Total Probability

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Total Probability

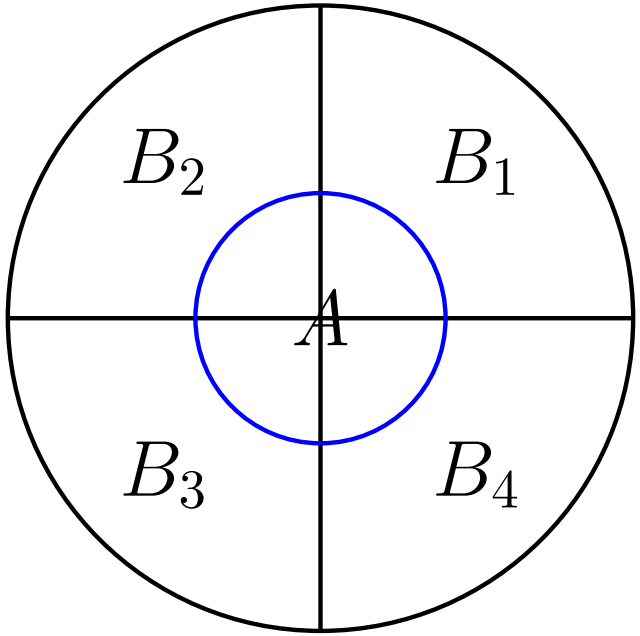
A set of events $B_1, B_2, \ldots, B_n$ is a **partition** of Ω if:

- They are pairwise disjoint: $B_i \cap B_j = \emptyset$ for any $i \neq j$.
- Their union equals Ω: $B_1 \cup B_2 \cup \ldots \cup B_n = \Omega$.

The **law of total probability** states:

$$p(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + \ldots + P(A|B_n)P(B_n)$$

Visualization Example



Special Case: Total Probability with Two Events

For any event B, B and B^c form a partition of Ω. The law of total probability simplifies to:

$$P(A) = P(A|B)P(B) + P(A|B^c)P(B^c)$$

In-Class Problem 1

Problem: Two urns scenario

- Urn 1: 4 black balls, 3 white balls.
- Urn 2: 2 black balls, 2 white balls.
- You randomly pick an urn and select a ball.

Question: What is the probability that the ball is white?

In-Class Problem 2

Problem: Power supply failure scenario

- Main power supply failure probability: 10%.
- Auxiliary power supply failure probabilities:
  - If main power is running: 10%.
  - If main power fails: 15% (due to overload).

Question: What is the probability that the auxiliary power will fail?