Localization in eigenvector centrality, and a fix

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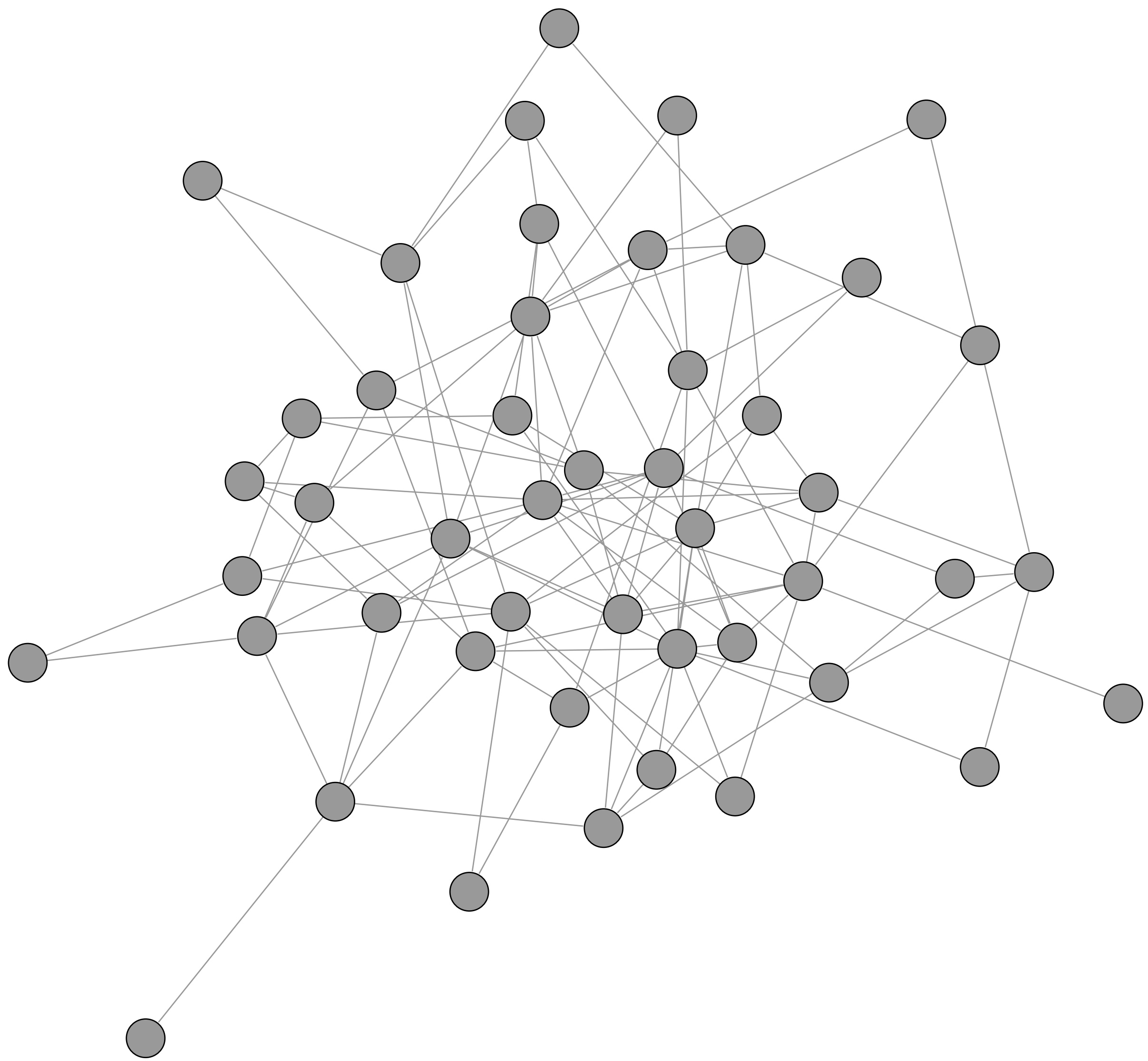
Which node is the most important?



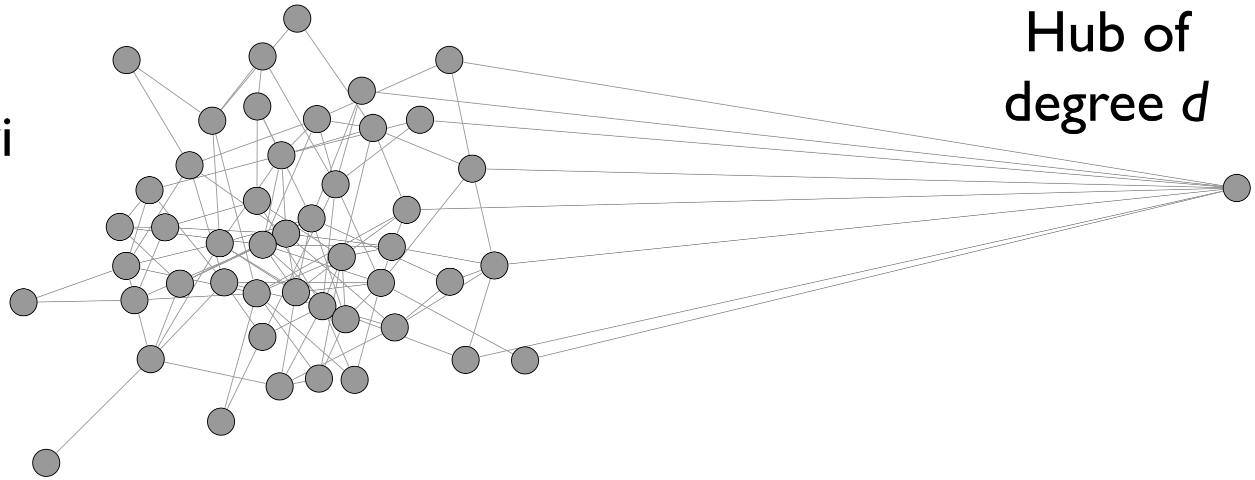
Eigenvector centrality can give unintuitive results

Centrality ***v*** is leading eigenvector of ***A***

$$v_i \propto \sum_j A_{ij} v_j$$

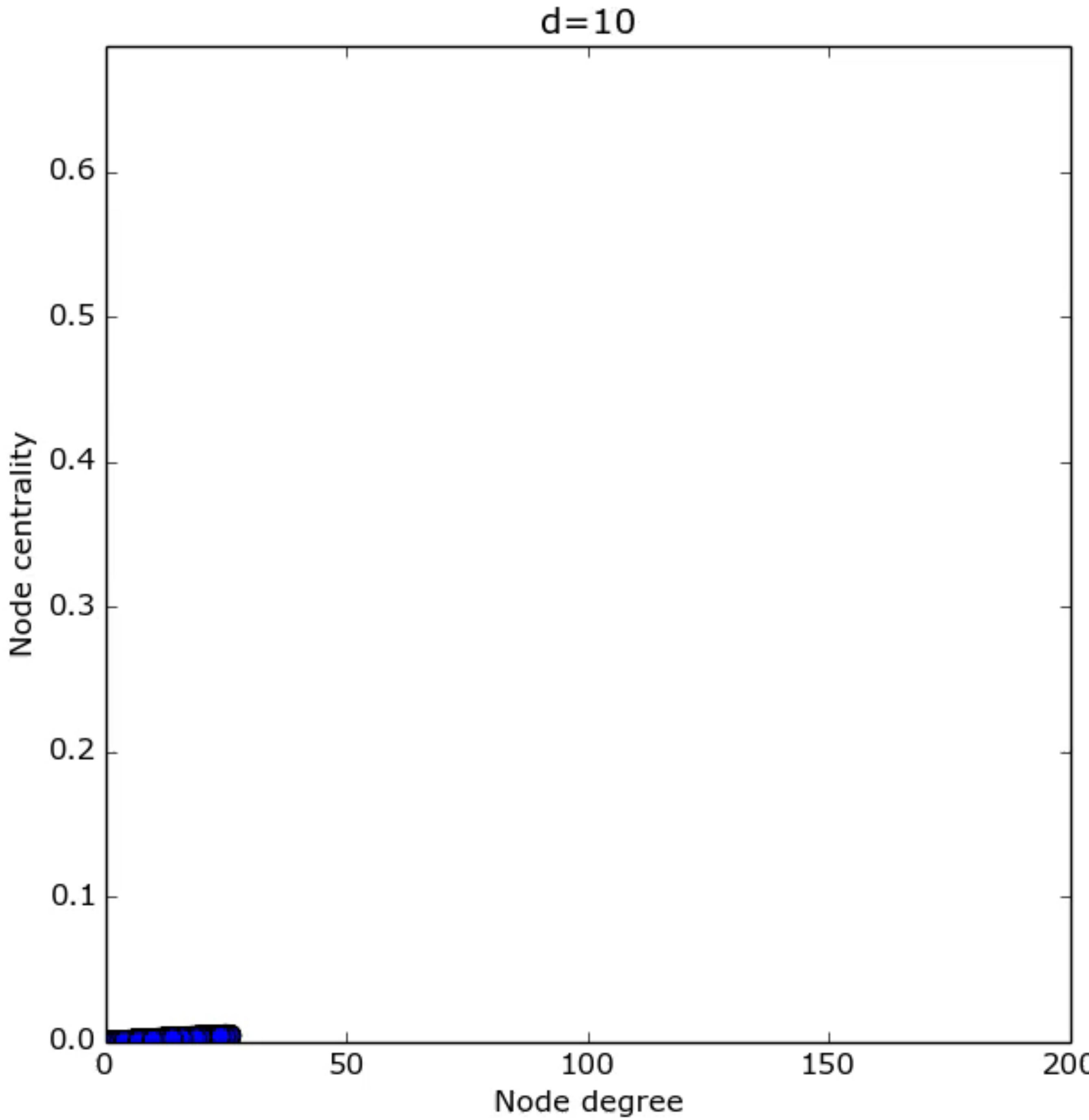
Model:

Erdős-Rényi
Size *n*
Degree *c*



Hub of degree *d*

n = 400,000
c = 10
varying d

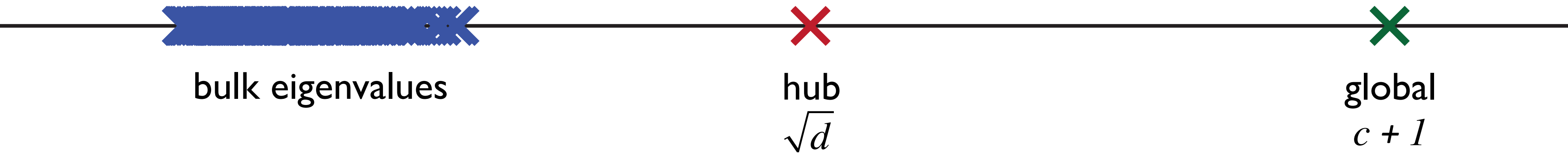


When and why does this occur?

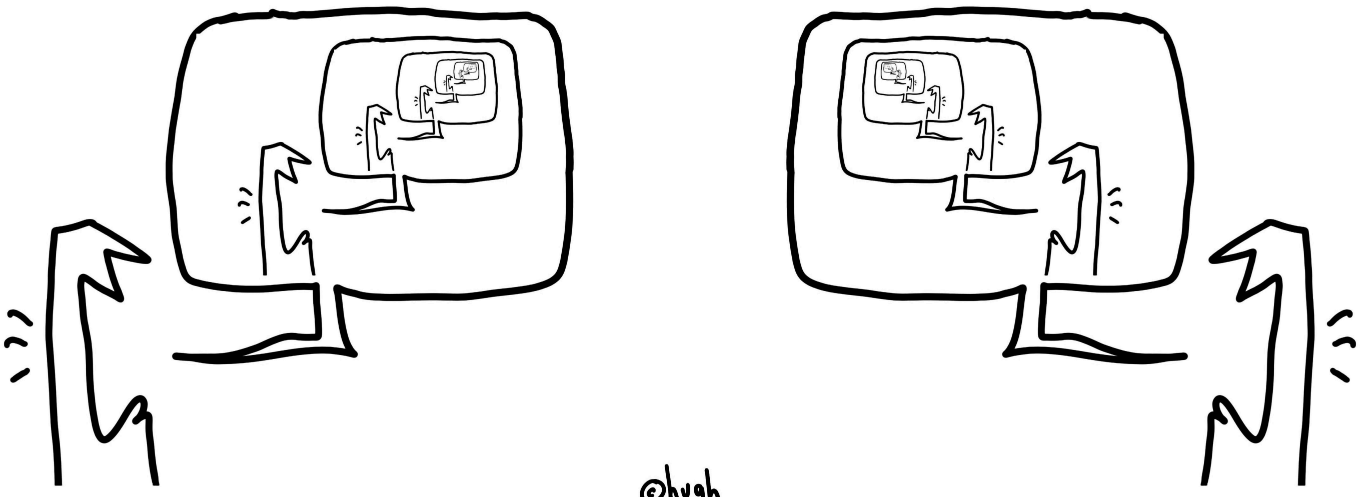
How can it be prevented?

Why does this pathology occur?

Underlying spectral transition

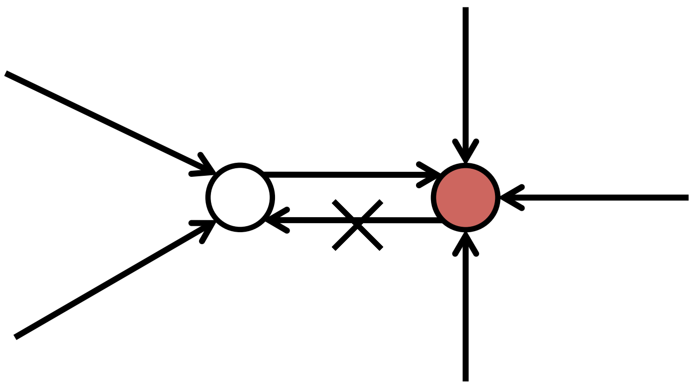


Echo chamber effect



http://www.gapingvoid.com/echochamber123.jpg

Non-backtracking centrality prevents the echo chamber

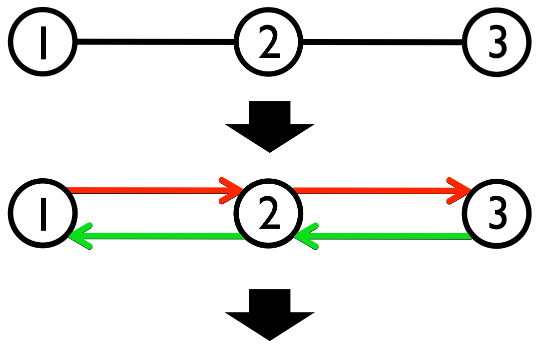


Very similar to eigenvector centrality

- Asymptotically equivalent for dense networks
- Avoids hub localization on sparse networks

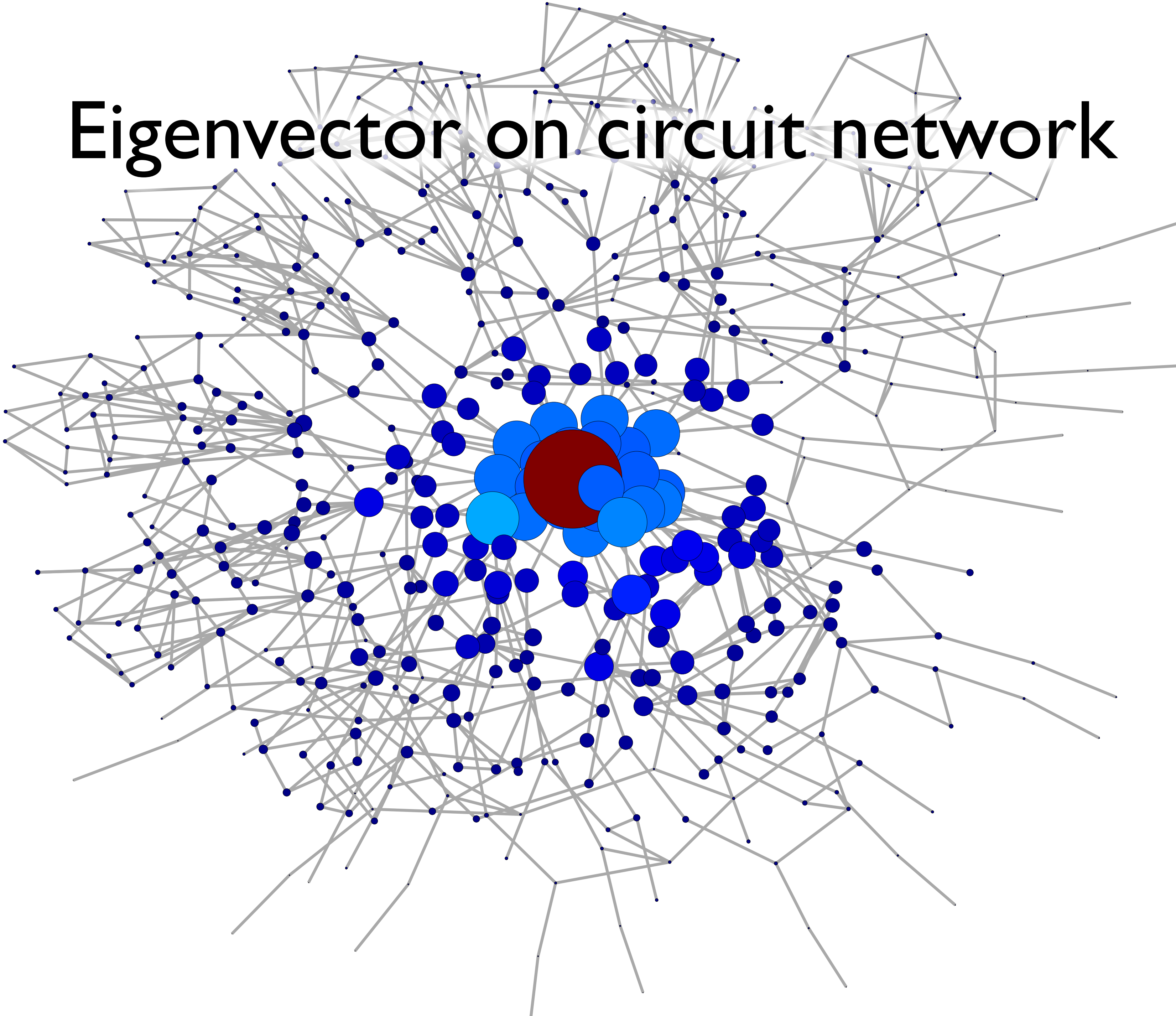
Constructing *B*

$$B_{i\to j,k\to l} = \delta_{il}(1-\delta_{jk})$$

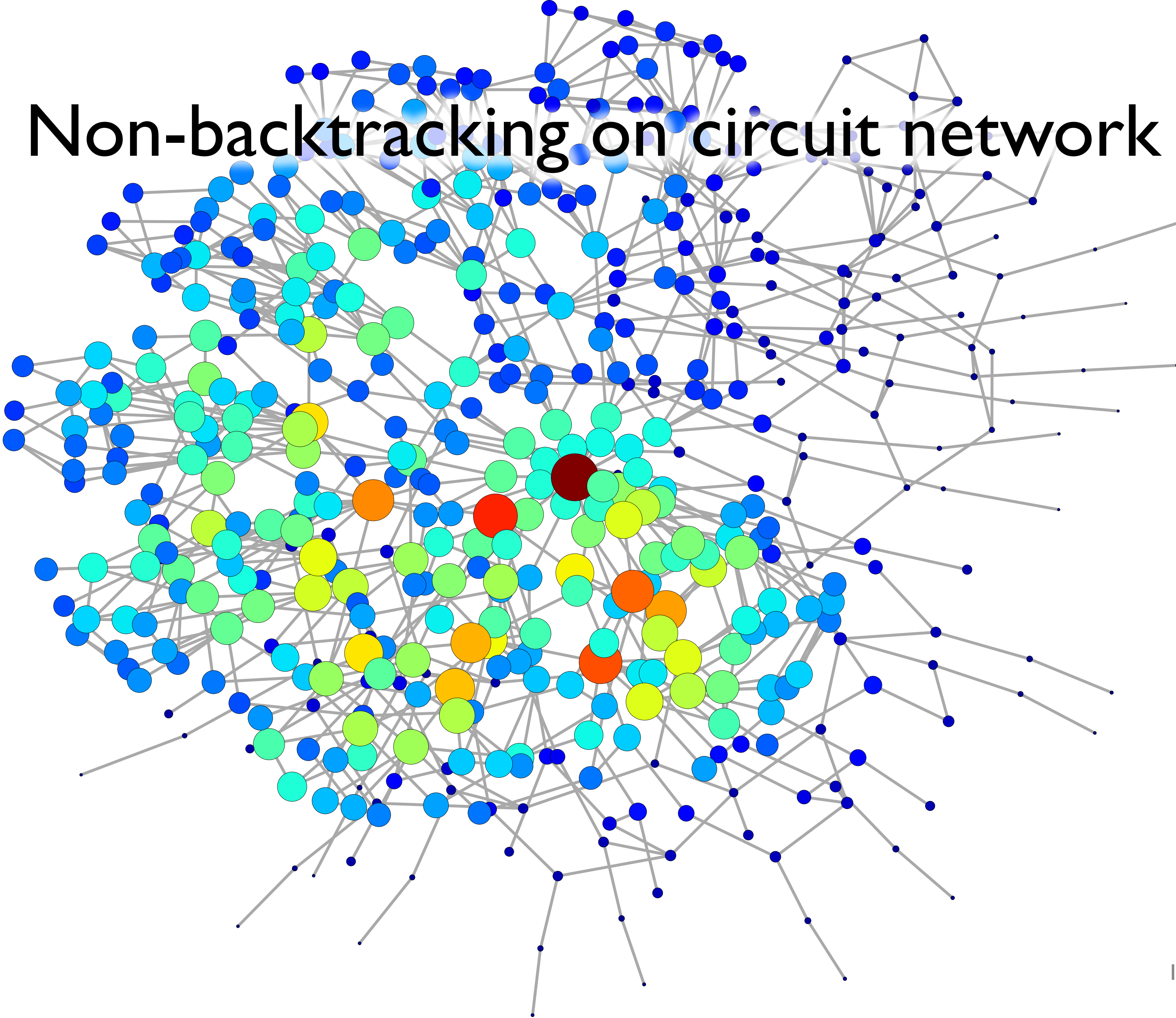


	1→2	2→1	2→3	3→2
1→2	0	0	0	0
2→1	0	0	0	1
2→3	1	0	0	0
3→2	0	0	0	0

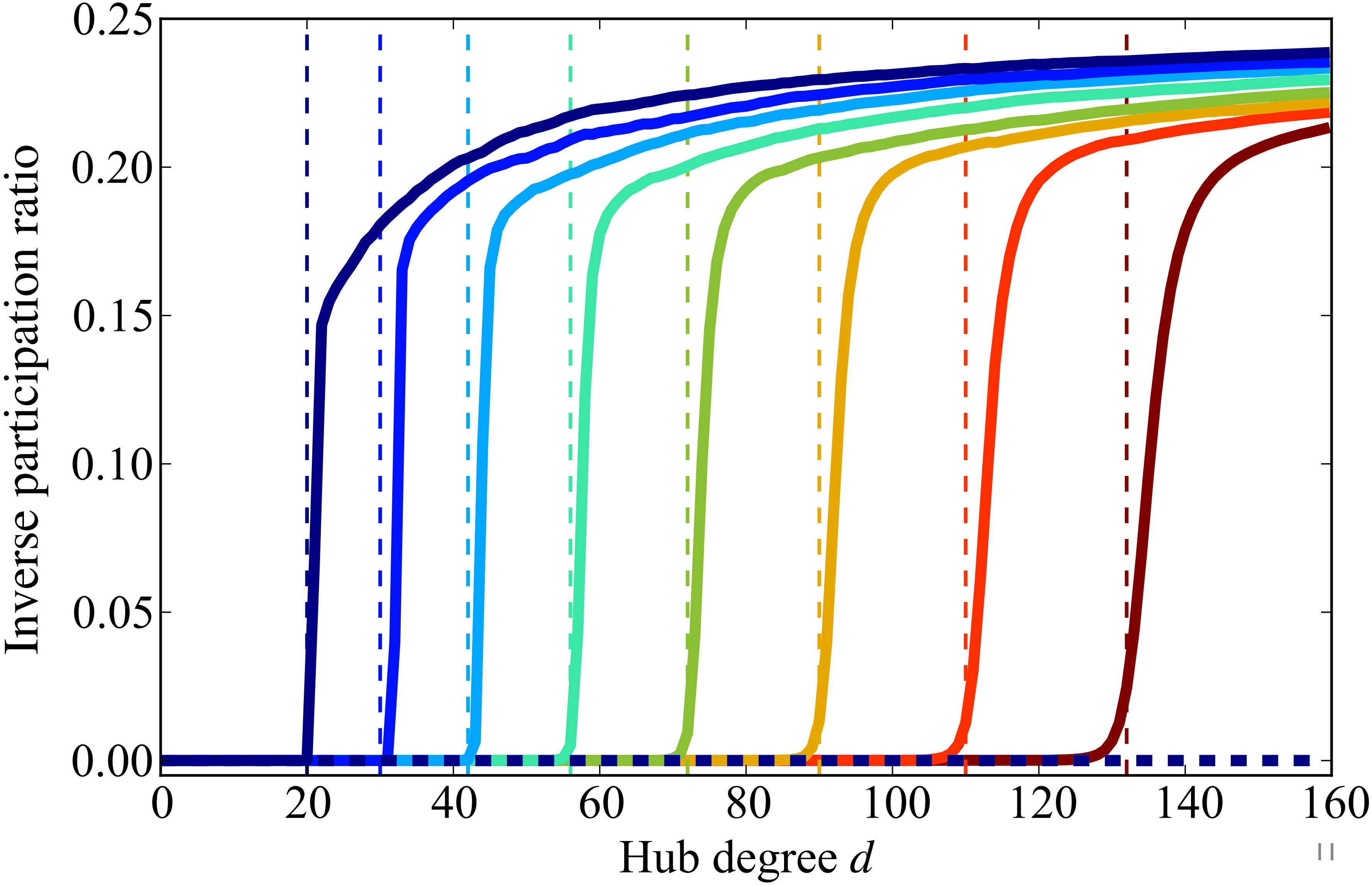
Eigenvector on circuit network



Non-backtracking on circuit network



Synthetic results agree with theory



Inverse participation ratio for a variety of networks

	Network	Nodes	Eigenvector	Non-backtracking
Synthetic	Planted hub, $d = 70$	1 000 001	2.6×10^{-6}	1.4×10^{-6}
	Planted hub, $d = 120$	1 000 001	0.2567	1.4×10^{-6}
	Power law, $\alpha = 2.1$	1 000 000	0.0089	0.0040
	Power law, $\alpha = 2.9$	1 000 000	0.2548	0.0011
Empirical	Physics collaboration	12 008	0.0039	0.0039
	Word associations	13 356	0.0305	0.0075
	Youtube friendships	1 138 499	0.0479	0.0047
	Company ownership	7 253	0.2504	0.0161
	Ph.D. advising	1 882	0.2511	0.0386
	Electronic circuit	512	0.1792	0.0056
	Amazon	334 863	0.0510	0.0339

In Summary

Non-backtracking centrality

– Asymptotically equivalent to eigenvector centrality in dense networks

– Prevents localization due to hubs

– Can be calculated efficiently

But

– Doesn’t prevent all types of localization

– Doesn’t work well on trees

– Other measures aren’t susceptible to localization

Localization and centrality on networks
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Thanks!