Camera Pose Estimation and RANSAC

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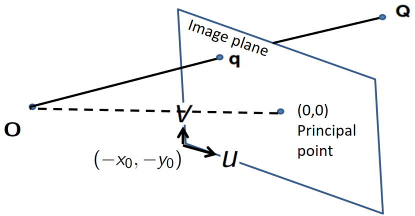
Presentation Outline

Camera Models and Projection (Reminder)



- Let the optical center be the origin of the camera.
- Let (X^m, Y^m, Z^m) be the coordinates of a 3D point $\mathbf{Q}$, relative to the world system.
- Let the 2D pixel be denoted by $\mathbf{q}(u, v, 1)^T$.

Camera Models and Projection (Reminder)

- Projection of 3D point on the image:

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \sim \begin{pmatrix} \mathrm{K} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathrm{R} & -\mathrm{R}\mathbf{t} \\ \mathbf{0}^T & 1 \end{pmatrix} \begin{pmatrix} X^m \\ Y^m \\ Z^m \\ 1 \end{pmatrix}$$

- The following 3×3 matrix is the camera matrix:

$$\mathrm{K} = \begin{pmatrix} k_u f & 0 & k_u x_0 \\ 0 & k_v f & k_v y_0 \\ 0 & 0 & 1 \end{pmatrix}$$

Projection Matrix (Reminder)

- The projection matrix that maps 3D points to 2D image is given by:

$$P = \begin{pmatrix} K & \mathbf{0} \end{pmatrix} \begin{pmatrix} R & -R\mathbf{t} \\ \mathbf{0}^T & 1 \end{pmatrix}$$

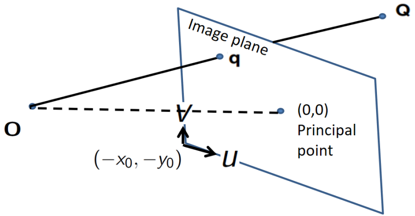
$$P = \begin{pmatrix} KR & -KR\mathbf{t} \end{pmatrix}$$

$$P = KR \begin{pmatrix} I & -\mathbf{t} \end{pmatrix}$$

What is Camera Calibration?

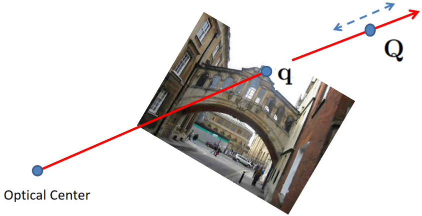
- The task refers to the problem of computing the calibration matrix K.
- In other words, we compute the focal length, principal point, and aspect ratio in the camera matrix.

Forward Projection



$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \sim \mathrm{KR} \begin{pmatrix} \mathbf{I} & -\mathbf{t} \end{pmatrix} \begin{pmatrix} X^m \\ Y^m \\ Z^m \\ 1 \end{pmatrix}$$

Backward Projection



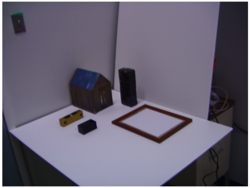
$$\mathbf{Q} \sim \mathsf{K}^{-1}\mathbf{q}$$

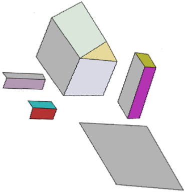
$$\mathbf{Q} \sim \mathsf{K}^{-1} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$

Presentation Outline

What is pose estimation?

The problem of determining the position and orientation of the camera relative to the object (or vice-versa).

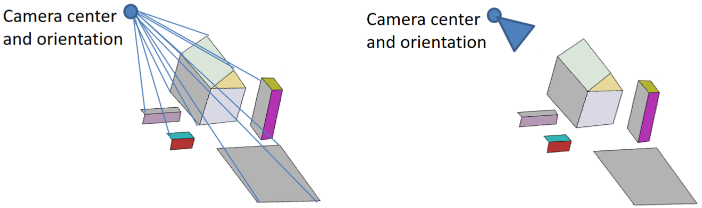




Left: Camera Image, Right: 3D model of the world

What is pose estimation?

The problem of determining the position and orientation of the camera relative to the object (or vice-versa).



We use the correspondences between 2D image pixels (and thus camera rays) and 3D object points (from the world) to compute the pose.

Pose Estimation

- We consider that the camera is calibrated, i.e. we know its calibration matrix K.
- We are given three 2D image to 3D object correspondences. Let the 3 2D points be given by:

$$\mathbf{q_1} = \begin{pmatrix} u_1 \\ v_1 \\ 1 \end{pmatrix} \quad \mathbf{q_2} = \begin{pmatrix} u_2 \\ v_2 \\ 1 \end{pmatrix} \quad \mathbf{q_3} = \begin{pmatrix} u_3 \\ v_3 \\ 1 \end{pmatrix}.$$

- Let the 3 3D points be given by:

$$\mathbf{Q_1^m}, \mathbf{Q_2^m}, \mathbf{Q_3^m}$$

Input and Unknowns

Given $\mathbf{q_i}, \mathbf{Q_i^m}, i = \{1, 2, 3\}$, and K in the following equation:

$$\mathbf{q_i} = \mathrm{K}\mathbf{R} \begin{pmatrix} \mathbf{I} & -\mathbf{t} \end{pmatrix} \mathbf{Q_i^m}, i = \{1, 2, 3\}$$

Our goal is to compute the rotation matrix R and the translation $\mathbf{t}$.

Pairwise Distance Computation

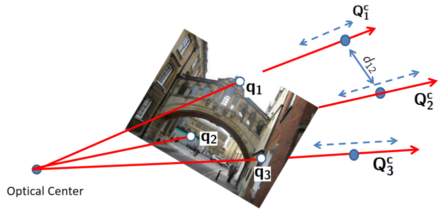
- Given the three 3D points $\mathbf{Q_i^m}, i = \{1, 2, 3\}$ we compute the 3 pairwise distances d_{12}, d_{23}, and d_{31} as follows:
  - $$d_{ij} = dist(\mathbf{Q_i^m}, \mathbf{Q_j^m})$$
  - $$dist(\mathbf{Q_i^m}, \mathbf{Q_j^m}) = \sqrt{(X_i^m - X_j^m)^2 + (Y_i^m - Y_j^m)^2 + (Z_i^m - Z_j^m)^2}$$

World frame to Camera frame

- Let the three 3D points $\mathbf{Q_i^m}, i = \{1, 2, 3\}$ be denoted by $\mathbf{Q_i^c}, i = \{1, 2, 3\}$ in the camera coordinate system.
- In other words, we have $\mathbf{Q_i^c} = R\mathbf{Q_i^m} - R\mathbf{t}$.
- Here $\mathbf{Q_i^m}$'s are known variables and $\mathbf{Q_i^c}$'s are unknowns.
- It is easy to observe the following since the distance between two points do not change when we transform them from one coordinate frame to another:

$$dist(\mathbf{Q_i^m}, \mathbf{Q_j^m}) = dist(\mathbf{Q_i^c}, \mathbf{Q_j^c})$$

Reformulation of Pose Estimation



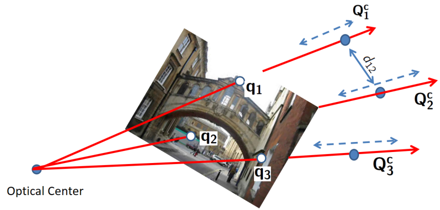
We can compute $\mathbf{Q_i^c}$ as follows:

$$\mathbf{Q_i^c} \sim K^{-1}\mathbf{q_i}$$

$$\mathbf{Q_i^c} = \lambda_i K^{-1}\mathbf{q_i}$$

Here λ_i is an unknown scalar that determines the distance of the 3D point $\mathbf{Q_i^c}$ from the optical center along the ray $\mathbf{OQ_i^c}$.

Reformulation of Pose Estimation

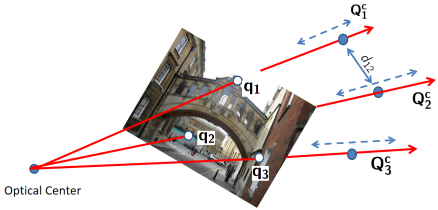


$$\mathbf{Q_i^c} = \lambda_i \mathsf{K}^{-1}\mathbf{q_i}$$

We simplify the notations, let us denote $\mathsf{K}^{-1}\mathbf{q_i}$ as follows:

$$\mathsf{K}^{-1}\mathbf{q_i} = \begin{pmatrix} X_i \\ Y_i \\ Z_i \end{pmatrix} \quad (1)$$

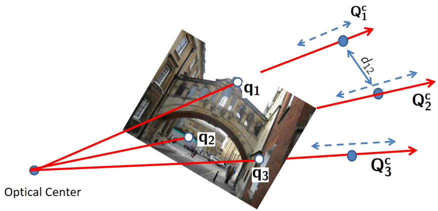
Reformulation of Pose Estimation



$$\mathbf{Q_i^c} = \lambda_i \begin{pmatrix} X_i \\ Y_i \\ Z_i \end{pmatrix}$$

The pose estimation can be seen as the computation of the unknown λ_i parameters.

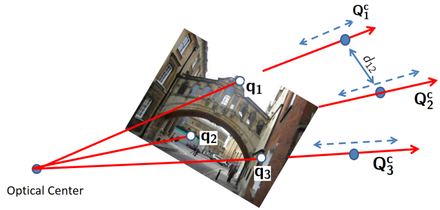
Reformulation of Pose Estimation



$$dist(\mathbf{Q_i^c}, \mathbf{Q_j^c}) = dist(\mathbf{Q_i^m}, \mathbf{Q_j^m}) = d_{ij}, \forall i, j = \{1, 2, 3\}, i \neq j$$

$$\sqrt{(\lambda_i X_i - \lambda_j X_j)^2 + (\lambda_i Y_i - \lambda_j Y_j)^2 + (\lambda_i Z_i - \lambda_j Z_j)^2} = d_{ij}$$

Reformulation of Pose Estimation



$$
\begin{aligned}
(\lambda_1 X_1 - \lambda_2 X_2)^2 + (\lambda_1 Y_1 - \lambda_2 Y_2)^2 + (\lambda_1 Z_1 - \lambda_2 Z_2)^2 &= d_{12}^2 \\
(\lambda_2 X_2 - \lambda_3 X_3)^2 + (\lambda_2 Y_3 - \lambda_3 Y_3)^2 + (\lambda_2 Z_2 - \lambda_3 Z_3)^2 &= d_{23}^2 \\
(\lambda_3 X_3 - \lambda_1 X_1)^2 + (\lambda_3 Y_3 - \lambda_1 Y_1)^2 + (\lambda_3 Z_3 - \lambda_1 Z_1)^2 &= d_{31}^2
\end{aligned}
$$

We have 3 quadratic equations and 3 unknowns.

Reformulation of Pose Estimation

$$
\begin{aligned}
(\lambda_1 X_1 - \lambda_2 X_2)^2 + (\lambda_1 Y_1 - \lambda_2 Y_2)^2 + (\lambda_1 Z_1 - \lambda_2 Z_2)^2 &= d_{12}^2 \\
(\lambda_2 X_2 - \lambda_3 X_3)^2 + (\lambda_2 Y_3 - \lambda_3 Y_3)^2 + (\lambda_2 Z_2 - \lambda_3 Z_3)^2 &= d_{23}^2 \\
(\lambda_3 X_3 - \lambda_1 X_1)^2 + (\lambda_3 Y_3 - \lambda_1 Y_1)^2 + (\lambda_3 Z_3 - \lambda_1 Z_1)^2 &= d_{31}^2
\end{aligned}
$$

- We have 3 quadratic equations and 3 unknowns.
- We can have a total of 2^3 possible solutions for the three parameters $(\lambda_1, \lambda_2, \lambda_3)$.
- Several numerical methods exist to solve the polynomial system of equations.

How to identify a unique solution?

- Out of the 8 solutions, only one will be the correct solution.
- In some of the solutions, the 3D point will be behind the camera.
- Using additional point correspondence, we can identify the correct solution.

Computing the Pose

- We remind you the relation between $\mathbf{Q}_i^c$ and $\mathbf{Q}_i^m$: $\mathbf{Q}_i^c = R\mathbf{Q}_i^m - R\mathbf{t}$.
- We are given $\mathbf{Q}_i^m$ and we have computed $\mathbf{Q}_i^c$.
- From three 3D-to-3D point correspondences we can compute the transformation parameters $(R, \mathbf{t})$ using Horn's method.

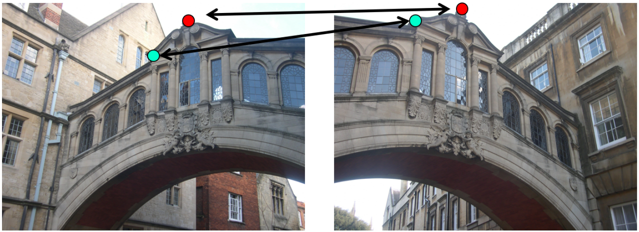
Presentation Outline

Matching Images

Matching Images

We match keypoints from left and right images.

Matching Images



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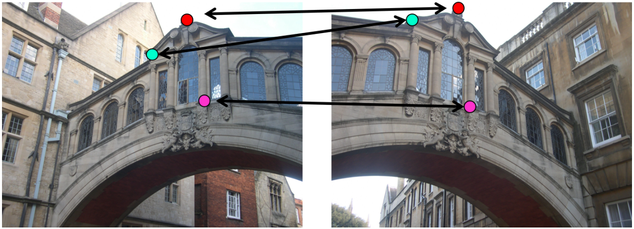

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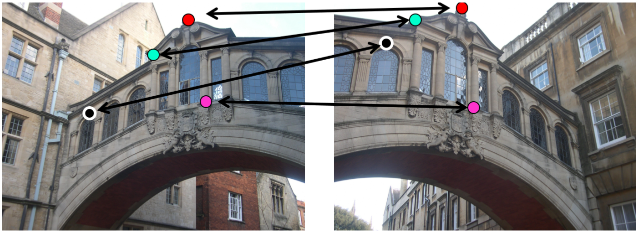

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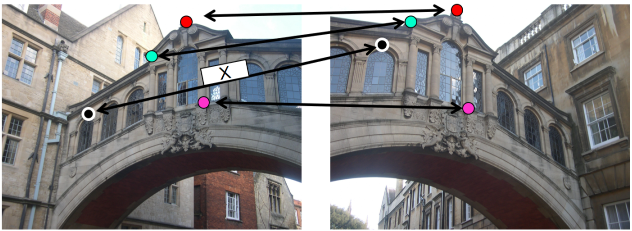

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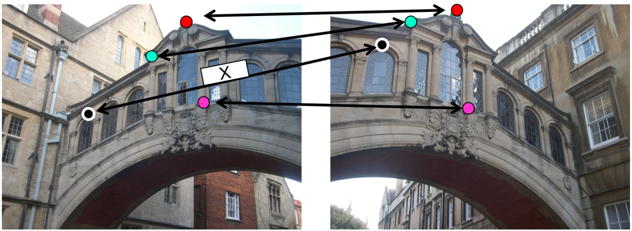
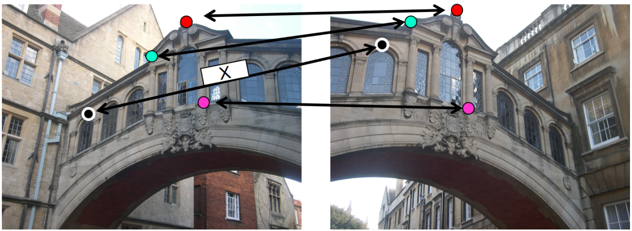
Matching Images



We match keypoints from left and right images.

- One of the matches is incorrect!

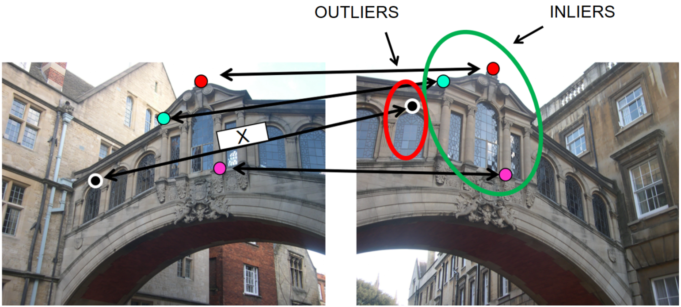
Matching Images



We match keypoints from left and right images.

- One of the matches is incorrect!
- In a general image matching problem, we can have 100's of incorrect matches.

Outliers and Inliers



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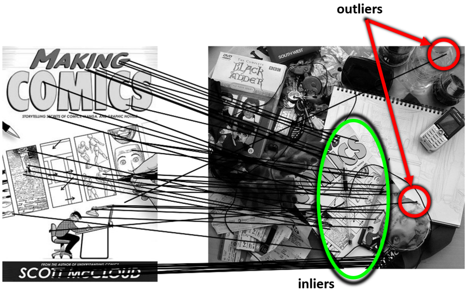
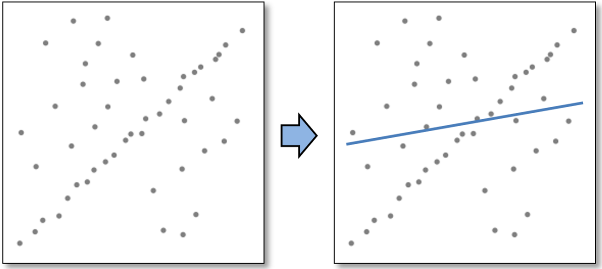
Outliers and Inliers



We match keypoints from left and right images.

Robustness

- Lets consider a simpler example linear regression.



Problem: Fit a line to these datapoints

Least squares fit

- How can we fix this?

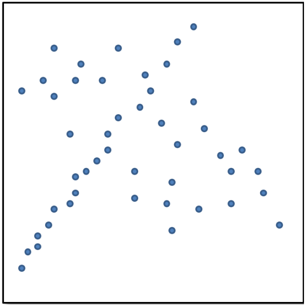
Slide: Noah Snavely

Idea

- Given a hypothesized line.
- Count the number of points that agree with the line, i.e., points within a small distance of the line.
- For all possible lines, select the one with the largest number of inliers.

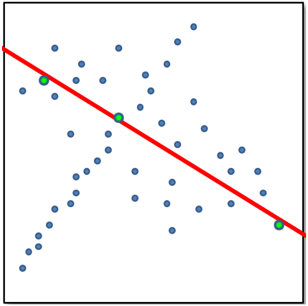
Slide: Noah Snavely

Counting Inliers



Slide: Noah Snavely

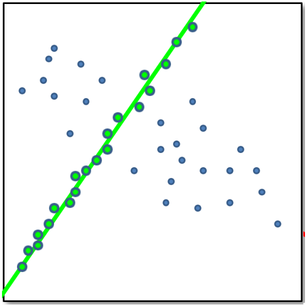
Counting Inliers



- 3 inliers

Slide: Noah Snavely

Counting Inliers



- 20 inliers!

Slide: Noah Snavely

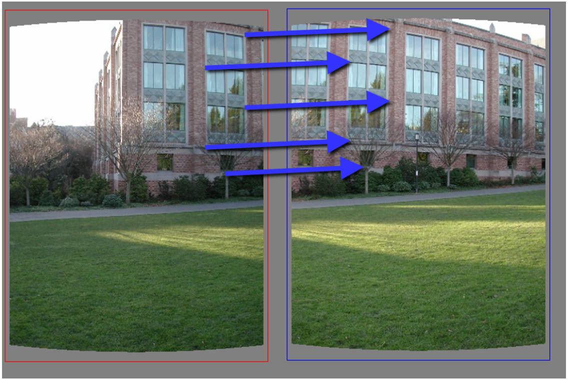
How do we find the best line?

- Unlike least-squares, no simple closed-form solution
- Hypothesize-and-test
  - Try out many lines, keep the best one
  - Which lines?

Translations

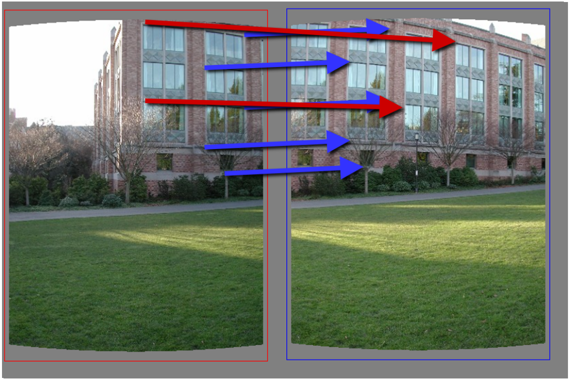
Slide: Noah Snavely

RANdom SAmple Consensus



Slide: Noah Snavely

RANdom SAmple Consensus



Slide: Noah Snavely

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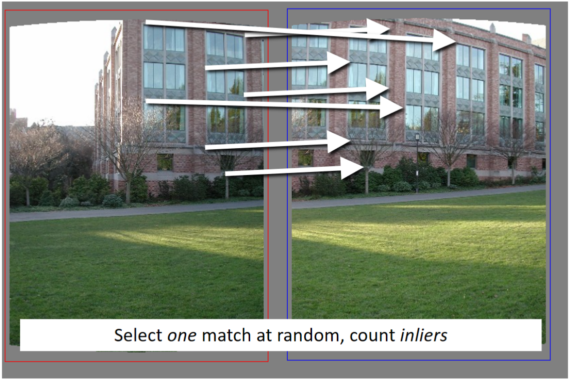

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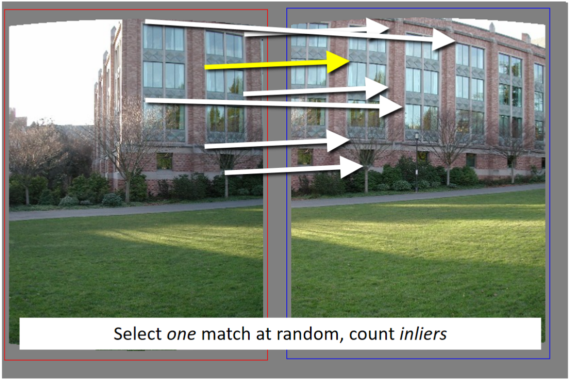
RANdom SAmple Consensus



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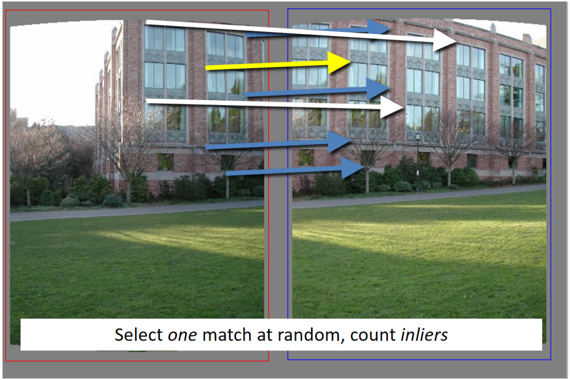

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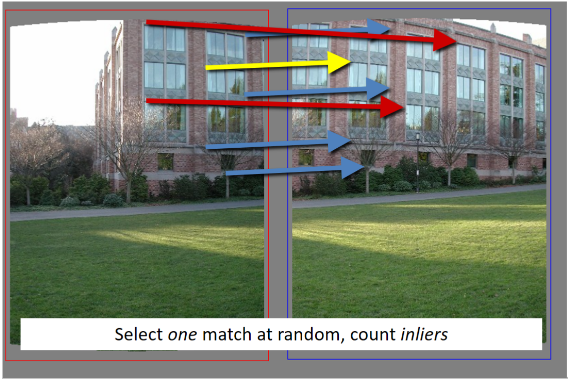

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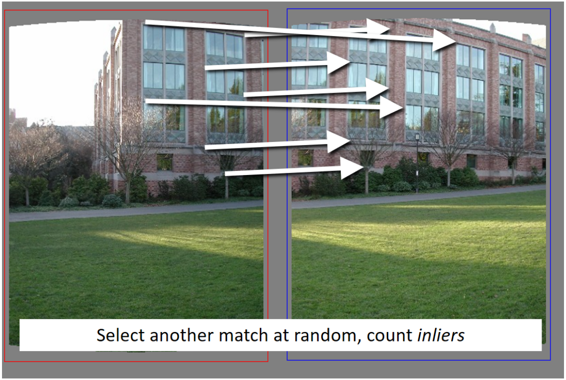

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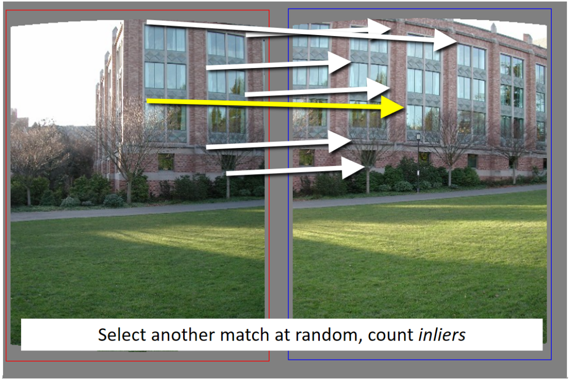

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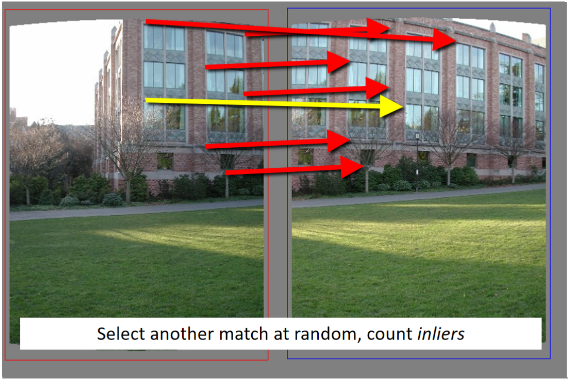

Slide: Noah Snavely

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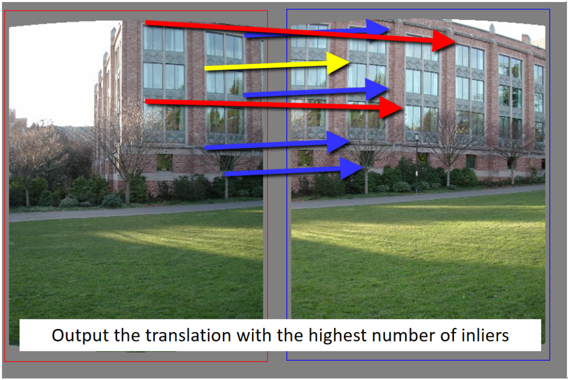


Slide: Noah Snavely

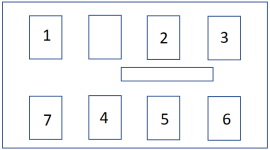
RANSAC

Idea:

- All the inliers will agree with each other on the translation vector; the (hopefully small) number of outliers will (hopefully) disagree with each other
  - RANSAC only has guarantees if there are $\leq$ 50% outliers
- All good matches are alike; every bad match is bad in its own way - Alyosha Efros, CMU

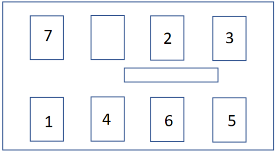
Slide: Noah Snavely

Pose Estimation



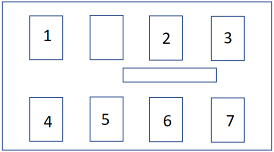
- Inliers?

Pose Estimation



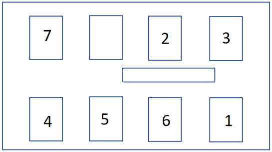
- Inliers?

Pose Estimation



- Inliers?

Pose Estimation

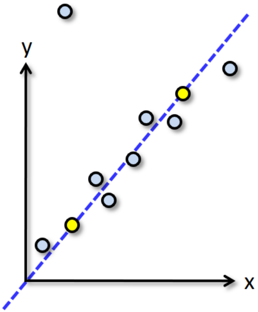


- Inliers?

RANSAC

- **Inlier threshold** related to the amount of noise we expect in inliers
  - Often model noise as Gaussian with some standard deviation (e.g., 3 pixels)
- **Number of rounds** related to the percentage of outliers we expect, and the probability of success we would like to guarantee
  - Suppose there are 20% outliers, and we want to find the correct answer with 99% probability
  - How many rounds do we need?

Sample size



- How do we generate a hypothesis?

Slide: Noah Snavely

General Version - RANSAC

1. Randomly choose s samples
   - Typically s = minimum sample size that lets you fit a model
2. Fit a model (e.g., line) to those samples
3. Count the number of inliers that approximately fit the model
4. Repeat N times
5. Choose the model that has the largest set of inliers

Slide: Noah Snavely

How many rounds?

	proportion of outliers e						
s	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

$p = 0.99$

- If we have to choose s samples each time
  - with an outlier ratio e
  - and we want the right answer with probability p

Slide: M. Pollefeys

Acknowledgments

Some presentation slides are adapted from the following materials:

- Peter Sturm, Some lecture notes on geometric computer vision (available online).
- Kristen Grauman's computer vision lecture slides
- Noah Snavely's computer vision lecture slides