

Assignment 4

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Instructions

Please type up your reports, generate a PDF, and send me the output. I highly recommend you use L^AT_EX to typeset your assignments. While there is a bit of a learning curve, it is worth learning. If not, consider using Lyx, a front-end to L^AT_EX. If neither of those work, feel free to use Word or Writer to type up your assignments. Email your assignment to vshankar@math.utah.edu. The deadline will be indicated on the class webpage. You will also submit your code. Make sure it can be run on at least a couple of machines, and make sure the code is well commented so I know you know what you're doing.

NOTE:

- You will use ONE programming/scripting language for your assignment, and send me only code that is relevant to the assignment, along with instructions to run the code.
- You will be graded on the quality of your write-up; the more clearly you explain things, the more points you get. Writing brusque text with no clarity will be severely penalized. Attempt to sound like a textbook!
- Make sure to clearly mark the question number and sub-question number.
- Plagiarism in the text and the code will not be tolerated. If you *must* use someone else's code for some part of your assignment, do so and cite it. You will not get a full score, but you will at least get partial credit.

Problems

1. (10 points) Use the Chebyshev spectral method and the BDF2 time-stepping scheme to discretize and solve the 1D heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + f(x, t)$ on the domain $[-1, 1]$ subject to the boundary conditions $u(-1) = u(1) = 0$. Ignore the forcing term for a minute.
 - (a) Run a temporal convergence study by fixing the grid spacing h at a *sufficiently* small value (what counts as sufficiently small?). For this study, vary Δt from 0.1 to $1e-3$. Produce a plot of relative error versus Δt .
 - (b) Run a spatial convergence study by fixing the timestep Δt at a sufficiently small value. For this study, vary h from 0.1 to $1e-3$. Produce a plot of relative error vs h .

To compute errors (both ℓ_2 and ℓ_∞), you will do something a little different. This is where the forcing term comes in. First, *prescribe* a solution $u(x, t)$ that satisfies the above boundary conditions. Then, plug this into the PDE to get the forcing term $f(x, t)$ for all time. Now, as long as you use this $f(x, t)$ when discretizing the PDE, you are guaranteed that $u(x, t)$ is the exact solution. This is called the method of manufactured solutions. Use this method to compute exact solutions on all grids and measure convergence.

You will have to decide how to time-step the forcing term. You either do it explicitly or implicitly. Since it is known for all time, it should be straightforward to do it implicitly with the BDF2 scheme, but I leave this to you. If you do it explicitly, you will use the SBDF2 scheme (of course).