

Assignment 3

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March 6, 2016

Instructions

Please type up your reports, generate a PDF, and send me the output. I highly recommend you use L^AT_EX to typeset your assignments. While there is a bit of a learning curve, it is worth learning. If not, consider using Lyx, a front-end to L^AT_EX. If neither of those work, feel free to use Word or Writer to type up your assignments. Email your assignment to vshankar@math.utah.edu. The deadline will be indicated on the class webpage. You will also submit your code. Make sure it can be run on at least a couple of machines, and make sure the code is well commented so I know you know what you're doing.

NOTE:

- You will use ONE programming/scripting language for your assignment, and send me only code that is relevant to the assignment, along with instructions to run the code.
- You will be graded on the quality of your write-up; the more clearly you explain things, the more points you get. Writing brusque text with no clarity will be severely penalized. Attempt to sound like a textbook!
- Make sure to clearly mark the question number and sub-question number.
- Plagiarism in the text and the code will not be tolerated. If you *must* use someone else's code for some part of your assignment, do so and cite it. You will not get a full score, but you will at least get partial credit.

Problems

1. (10 points) Use second-order centered differences and Crank-Nicolson to discretize and solve the 2D heat equation on the domain $[0, 1] \times [0, 1]$. Assume zero Dirichlet boundary conditions on the boundary of the square. Let the initial condition be $u(\mathbf{x}, t) = \sin(\pi x) \cos(\pi y) e^{-\pi^2 t}$ evaluated at $t = 0$.
 - (a) Run a temporal convergence study by fixing the grid spacing h at a *sufficiently* small value (what counts as sufficiently small?). For this study, vary Δt from 0.1 to $1e-4$. Produce a plot of relative error versus Δt .
 - (b) Run a spatial convergence study by fixing the timestep Δt at a sufficiently small value. For this study, vary h from 0.1 to $1e-4$. Produce a plot of relative error vs h .

To compute errors (both ℓ_2 and ℓ_∞), compute an approximate solution U on a grid of width $h = 1e-5$ with $\Delta t = 1e-5$, and use that as your “gold” solution.

2. (5 points) Perform the Von Neumann stability analysis of the Crank-Nicolson scheme on the 2D heat equation. What can you conclude?