

Assignment 2

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Instructions

Please type up your reports, generate a PDF, and send me the output. I highly recommend you use L^AT_EX to typeset your assignments. While there is a bit of a learning curve, it is worth learning. If not, consider using Lyx, a front-end to L^AT_EX. If neither of those work, feel free to use Word or Writer to type up your assignments. Email your assignment to vshankar@math.utah.edu. The deadline will be indicated on the class webpage. You will also submit your code. Make sure it can be run on at least a couple of machines, and make sure the code is well commented so I know you know what you're doing.

NOTE:

- You will use ONE programming/scripting language for your assignment, and send me only code that is relevant to the assignment, along with instructions to run the code.
- You will be graded on the quality of your write-up; the more clearly you explain things, the more points you get. Writing brusque text with no clarity will be severely penalized. Attempt to sound like a textbook!
- Make sure to clearly mark the question number and sub-question number.
- Plagiarism in the text and the code will not be tolerated. If you *must* use someone else's code for some part of your assignment, do so and cite it. You will not get a full score, but you will at least get partial credit.

Problems

1. (2 points) Analyze the absolute stability of Backward Euler and the Trapezoidal Rule. Show that they are both A -stable.
2. (a) (2 points) Derive the classical RK4 method from Simpson's Rule.
(b) (1 point) Draw its Butcher tableau.
(c) How would you go about deriving a general class of RK4 methods? Explain.
(d) (10 points) EXTRA CREDIT: Show the derivation of the above for.
3. You have been supplied with Matlab code for the Adams-Bashforth 2 method applied to the ODE IVP $\frac{dy}{dt} = -y^2, y(0) = 1$. However, the code opts for the easy route of starting up the AB2 method with the *exact* solution at time t_1 , given by $y(t_1) = \frac{1}{1+\Delta t}$. This is obviously not feasible in a real-world situation.
(a) Replace the use of the exact solution with one step of Forward Euler from t_0 to $t_{0+\Delta t}$.
(b) Replace the use of the exact solution with one RK2 step from t_0 to $t_{0+\Delta t}$; you may use any RK2 method.

- (c) Which approach is better in terms of accuracy, if any? Show results in the form of a plot to support your statement. This must be a convergence study with both RK2-bootstrapped-AB2 and FE-bootstrapped-AB2 on the same plot. Use a solution computed with a sufficiently small Δt as your “exact solution”, and compute the relative ℓ_2 error at a final time of your choice. Use a loglog scale for the plot.

NOTE: Carefully go over the test script that shows you how to use the code.

4. Using the AB2 code as a starting point and using the variable name conventions, write an AB3 code that is started up by a single step of RK3.
 - (a) Between AB2 and AB3, which would you use on a very stiff problem? Why?
 - (b) Between AB2 and AB3, which would you use on a problem with purely imaginary λ ? Why?
 - (c) Using a convergence study on the above function run out to a sufficiently long final time, demonstrate that AB3 converges an order faster than AB2. Use the same approach as in the previous question for the convergence study.