

Assignment 1

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Instructions

Please type up your replies, generate a PDF, and send me the output. I highly recommend you use L^AT_EX to typeset your assignments. While there is a bit of a learning curve, it is worth learning. If not, consider using Lyx, a front-end to L^AT_EX. If neither of those work, feel free to use Word or Writer to type up your assignments. Email your assignment to vshankar@math.utah.edu. The deadline will be indicated on the class webpage. You will also submit your code. Make sure it can be run on at least a couple of machines, and make sure the code is well commented so I know you know what you're doing.

Problems

1. Write code for generating polynomial interpolants by inverting the Vandermonde matrix (don't use libraries to do this). You will use both equispaced points on $[-1, 1]$ and Chebyshev *extrema* on $[-1, 1]$. You will be interpolating the Runge function $f(x) = \frac{1}{1+25x^2}$, assuming samples of the function are provided at your node set. Generate the following:
 - (a) A plot of condition number of the Vandermonde matrix vs number of nodes for both types of points.
 - (b) A plot of the polynomial interpolants for $N = 9$ and $N = 50$ for both types of points.
 - (c) Evaluate the Chebyshev extrema interpolant at 10,000 equispaced nodes. Produce a plot of total time (measured using Matlab's functions) vs the accuracy (measured as the ℓ_2 relative error in the polynomial interpolant) *only* for Chebyshev extrema.
 - (d) You have also been provided code to perform barycentric Lagrange interpolation at Chebyshev extrema. Produce a plot of total time vs accuracy (again measured as relative ℓ_2 error) for the Chebyshev extrema interpolant at 10,000 equispaced nodes.

Note that the relative ℓ_2 error is measured as $e_{\ell_2} = \frac{\|\mathbf{f} - \mathbf{p}\|_2}{\|\mathbf{f}\|_2}$, where $\mathbf{f} = [f(x_0), \dots, f(x_{10000})]^T$ and $\mathbf{p} = [p(x_0), \dots, p(x_{10000})]^T$.

2. Generate an approximation to the second derivative of some function $f(x)$ at 3 points, x_0, x_1 and x_2 using the polynomial interpolation approach at the point x_1 . Derive the error term using the error for polynomial interpolation; show the error for equispaced points with spacing h . Also give an error estimate for non-equally spaced points.
3. Generate an approximation to $\int_a^b f(x)dx$ using a quadratic polynomial interpolant to f . Assume you have 3 equispaced points, including the end points a and b ; that is, let $x_0 = a, x_1 = (a+b)/2$ and $x_2 = b$ be the 3 points. Derive the corresponding error term for this quadrature rule. What is this quadrature rule called?