Mid-Term 2

- Open book
- Open notes
- Everything through today
    - lexical scope, environments, closures, evaluation, assignment, parameter-passing mechanisms, types
- Example questions on the schedule page

HW9

New construct	*C equivalent*
`ref(x)`	`&x`
`setref(E1, E2)`	`(*E1 = E2, 1)`

```
let x = 0
 in let y = ref(x)
     in let d = setref(y, 2)
         in x
```

Result: **2**

HW9

```
let x = 0
 in let y = ref(x)
     in let d = setref(y, true)
         in x
```

Result: **`true`**

But should it be allowed?

HW9

```
let x = 0
 in let y = ref(x)
     in let d = if ...
                   then 1
                   else setref(y, true)
         in +(x, 0)
```

Might crash.

Solution: only allow assignments that do not change a variable's type

HW9

```
let x = 0 : int
  in let y = ref(x) : (refto int)
     in let d = setref(y, 1)
         in +(x, 0)
```

Ok

HW9

```
let x = 0 : int
  in let y = ref(x) : (refto int)
     in let d = setref(y, true)
         in +(x, 0)
```

Not ok

- First argument of **`setref`** must have type **`(refto T)`**
- Second argument of **`setref`** must have type **`T`**, for the same **`T`**

Back to our regularly scheduled programming...

`: squash`

Type-Checking Expressions

- What is the value of the following expression?

proc(x)+(x,1)

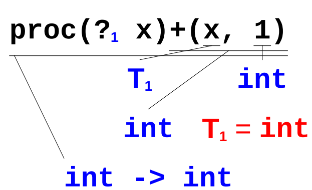
- **Answer:** Yet another trick question; it's not an expression in our typed language, because the argument type is missing
- But, clearly, the answer *should* be **(int -> int)**

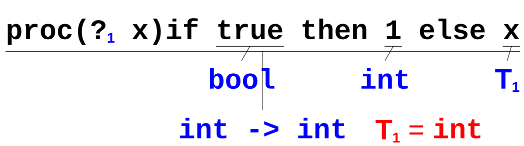
Type Inference

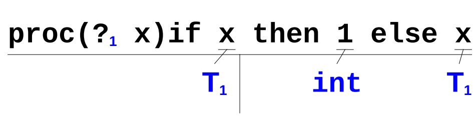
- ***Type inference*** is the process of inserting type annotations where the programmer omits them.
- We'll use explicit question marks, to make it clear where types are being omitted.

proc (?$_1$ x)+(x,1)

Type Inference

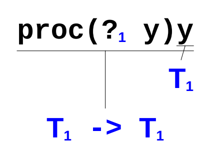


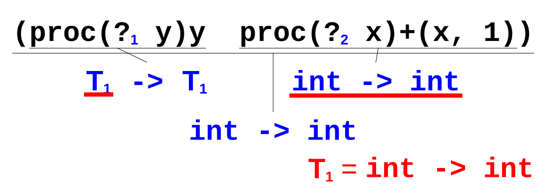


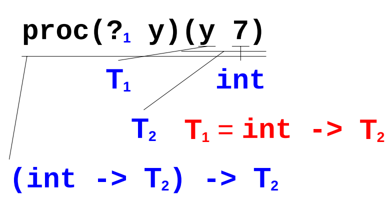


no type: T$_1$ can't be both **bool** and **int**

Type Inference







Type Inference

`proc(?`$_1$ **`x)(x x)`**

T_1 T_1

no type: T_1 can't be T_1 **`->`** ...

- T_1 can't be **`int`**
- T_1 can't be **`bool`**
- Suppose T_1 is T_2 **`->`** T_3
  - T_2 must be T_1
  - So we won't get anywhere!

Implementation

- Extend **`type`** datatype with **`tvar-type`** variant

```
(define-datatype type type?
  ...
  (tvar-type
    (serial-number integer?)
    (container vector?)))
```

- Create a new type variable record for each **`?`**
  - Initial container value is "don't know", **`'()`**
- Create a new type variable record for each application
- Change **`check-equal-type!`** to read and set type variable containers

The Universe of Programs

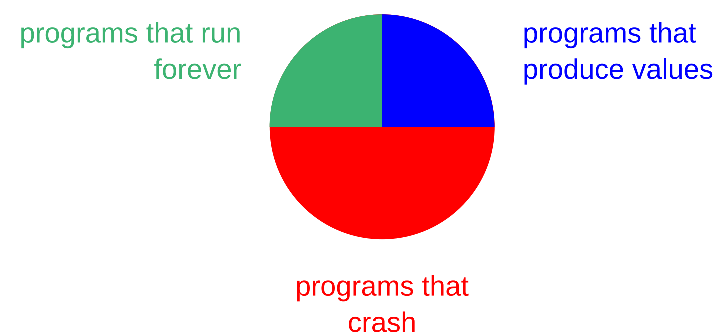
- The goal of type-checking is to rule out bad programs

```
+(1, true)
```

- Unfortunately, some good programs will be ruled out, too

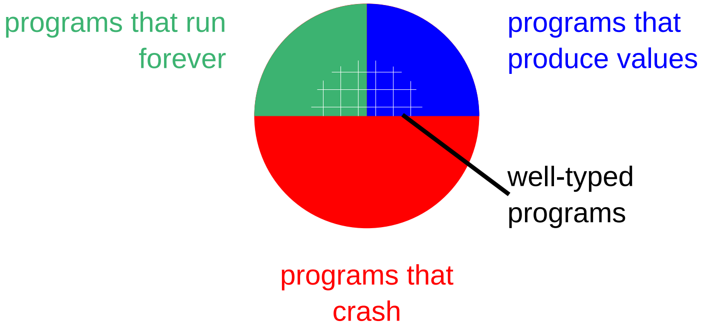
```
+(1, if true then 1 else false)
```

The Universe of Programs



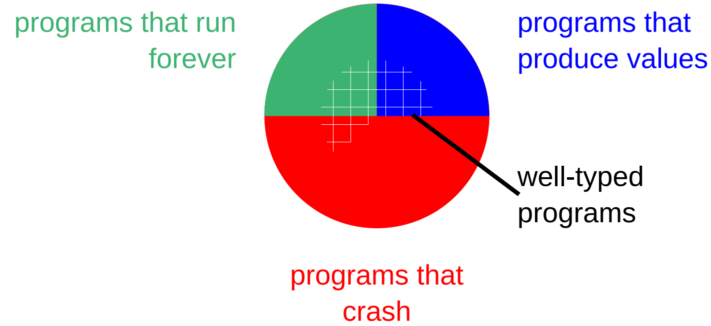
- Every program falls into one of three categories

The Universe of Programs



- The idea is that a type checker rules out the error category

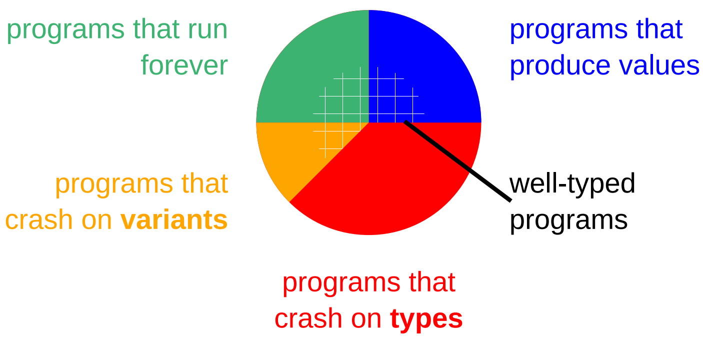
The Universe of Programs



- But a type checker for most languages will allow some errors!

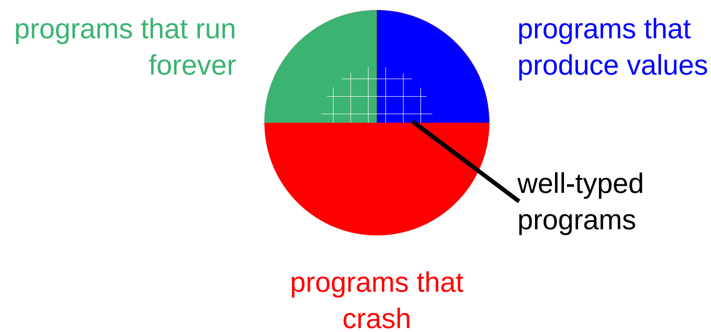
```
1 / 0 ⇒ divide by zero
```

The Universe of Programs



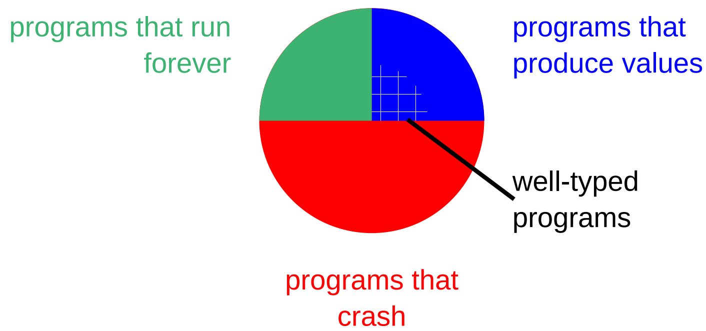
- Still, a type checker *always* rules out a certain class of errors
  - Division by 0 is a ***variant error***

The Universe of Programs



- Our language happens to have no variant errors, so the type checker rules out all errors

The Universe of Programs



- In fact, if we get rid of **`letrec`**, then every well-typed program terminates with a value!

Intution for Termination

Recall that to get rid of **`letrec`**

```
letrec int sum = proc(int x)
                  if zero?(x)
                     then 0
                     else +(x,(sum -(x, 1)))
  in (sum 10)
```

we can use self-application:

```
let sum = proc(int x, ? sum)
           if zero?(x)
              then 0
              else +(x,((sum sum) -(x, 1)))
  in ((sum sum) 10)
```

Intution for Termination

But we've already seen that we can't type self-application:

```
proc(?1 x)(x x)
        T1   T1
```

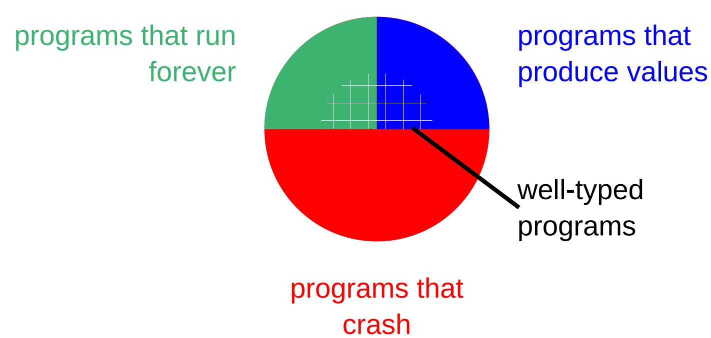
no type: T_1 can't be T_1 **`->`** ...

The only way around this restriction is to restore **`letrec`** or extend the type language.

(Extending the type language in this direction is beyond the scope of the course.)

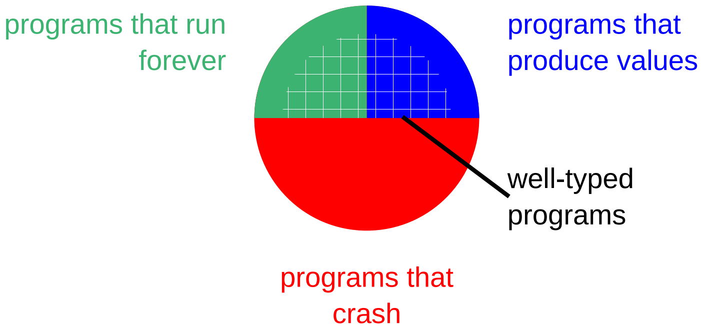
The Universe of Programs

- There are other ways that we'd like to expand the set of well-formed programs



The Universe of Programs

- There are other ways that we'd like to expand the set of well-formed programs



- Adjusting the type rules can allow more programs

Polymorphism

```
proc(?1 y)y
            T1
T1 -> T1
```

```
let f = prog(?1 y)y : T1 -> T1
 in if (f true) then (f 1) else (f 0)
 T1 -> T1     T1 -> T1     T1 -> T1
```

no type: T_1 can't be both **bool** and **int**

Polymorphism

- New rule: when type-checking the use of a let-bound variable, create fresh versions of unconstrained type variables

```
let f = prog(?1 y)y : T1 -> T1
 in if (f true) then (f 1) else (f 0)
 T2 -> T2     T3 -> T3     T4 -> T4
              int
```

T_2 = **bool** T_3 = **int** T_4 = **int**

- This rule is called ***let-based polymorphism***