Quiz

- Question #1: What is the value of the following expression?

```
+(1,1)
```

- Wrong answer: **0**.
- Wrong answer: **42**.

- Answer: **2**.

Quiz

- Question #2: What is the value of the following expression?

```
+ proc 8
```

- Wrong answer: **error**.

- Answer: Trick question! `+ proc 8` is not an expression.

Quiz

- Question #3: Is the following an expression?

```
add1(1, 7)
```

- Wrong answer: **No**.

- Answer: **Yes** (according to our grammar).

Quiz

- Question #4: What is the value of the following expression?

```
add1(1, 7)
```

- Answer: **2** (according to our interpreter).

- But no *real* language (e,g., C++) would accept **`add1(1, 7)`**.
- Let's agree to call **`add1(1, 7)`** an ***ill-formed expression*** because **`add1`** should be used with only one argument.
- Let's agree to never evaluate ill-formed expressions.

Quiz

- Question #5: What is the value of the following expression?

```
add1(1, 7)
```

- Answer: **None** - the expression is ill-formed.

Quiz

- Question #6: Is the following a well-formed expression?

```
+(proc(x)x, 5)
```

- Answer: **Yes**.

Quiz

- Question #7: What is the value of the following expression?

```
+(proc(x)x, 5)
```

- Answer: **None** - it produces an error:

> +: expects type <number> as 1st argument,
> given: (closure ((cbv-var x)) (var-exp x)
> (empty-env-record)); other arguments were: 5

- Let's agree that a **`proc`** expression cannot be inside a **`+`** form.

Quiz

- Question #8: Is the following a well-formed expression?

```
+(proc(x)x, 5)
```

- Answer: **No**.

Quiz

- Question #9: Is the following a well-formed expression?

```
+((proc(x)x 7), 5)
```

- Answer: Depends on what we meant by *inside* in our most recent agreement.
  - *Anywhere inside* - **No**.
  - *Immediately inside* - **Yes**.

- Since our intrepreter produces **12**, and since that result makes sense, let's agree on *immediately inside*.

Quiz

- Question #10: Is the following a well-formed expression?

```
+((proc(x)x true), 5)
```

- Answer: **Yes**, but we don't want it to be!

Quiz

- Question #11: Is it possible to define ***well-formed*** (as a decideable property) so that we reject all expressions that produce errors?

- Answer: **Yes**, obviously: reject *all* expressions!

Quiz

- Question #12: Is it possible to define ***well-formed*** (as a decideable property) so that we reject *only* expressions that produce errors?

- Answer: **No**.

```
+(1, if ... then 1 else proc(x)x)
```

- If we always knew whether `...` produces true or false, we could solve the halting problem.

Types

- Solution to our dilemma
  - In the process of rejecting expressions that are certainly bad, also reject some expressions that are good.

```
+(1, if (prime? 131101) then 1 else proc(x)x)
```

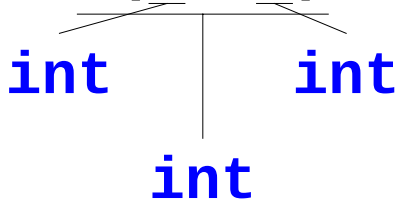
- Overall strategy:
  - Assign a ***type*** to each expression.
  - Compute the type of a complex expression based on the types of its subexpressions.

Types

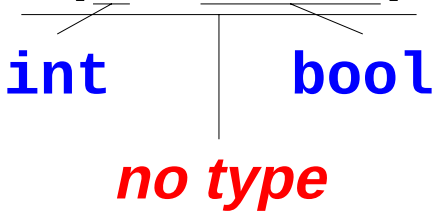
```
1 : int
```

```
true : bool
```

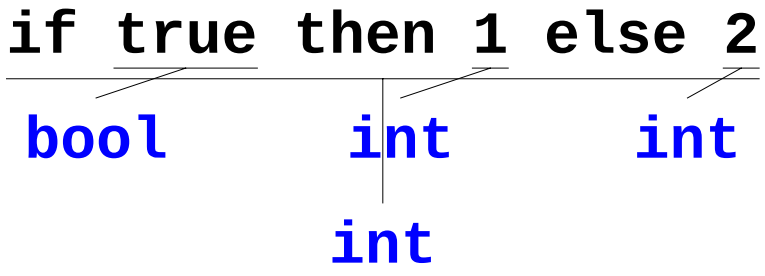
```
+(1, 2)
```

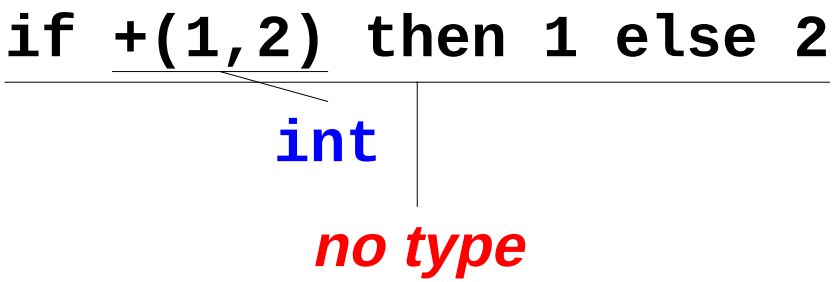


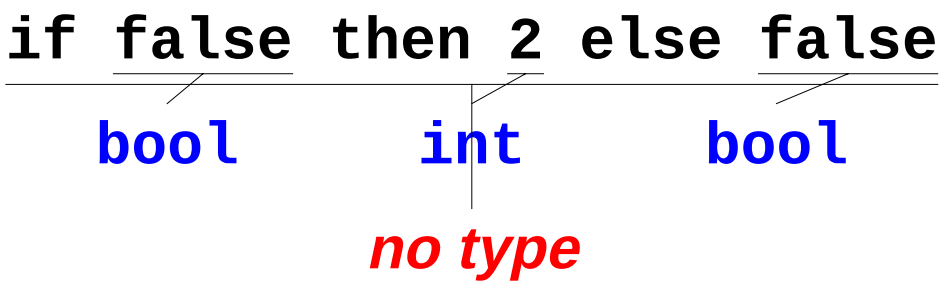

```
+(1, false)
```



Types


if true then 1 else 2
bool
int
int
int

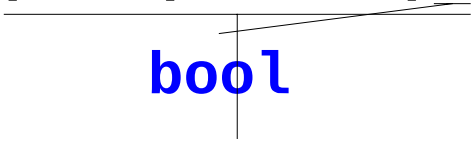

if +(1,2) then 1 else 2
int
no type


if false then 2 else false
bool
int
bool
no type

Types

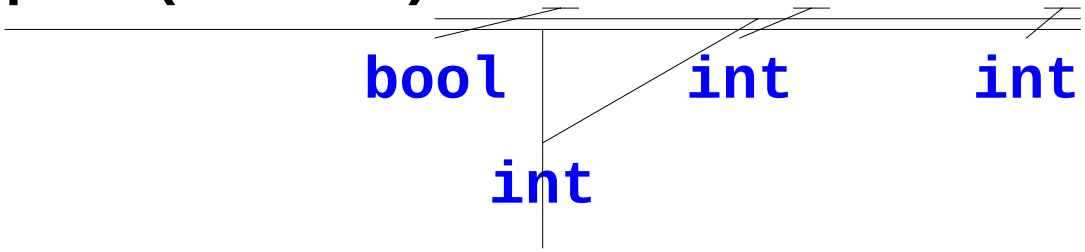
`x :` ***no type***

`proc(bool x)x`



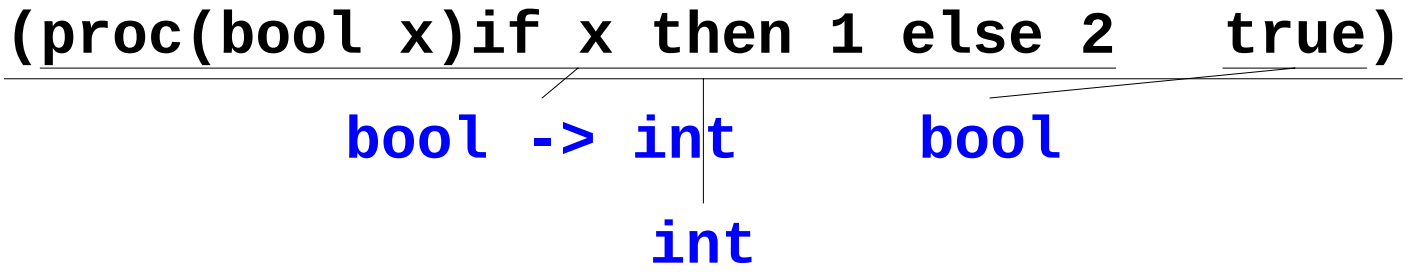
`bool -> bool`

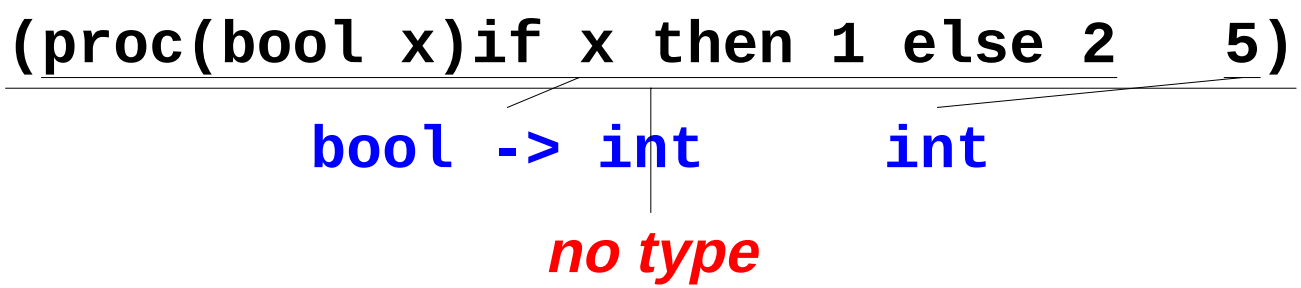
`proc(bool x)if x then 1 else 2`

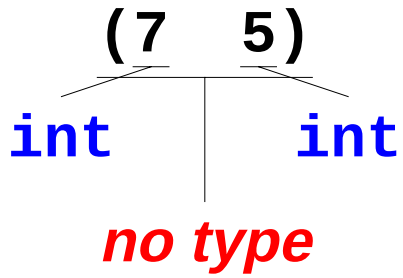


`bool -> int`

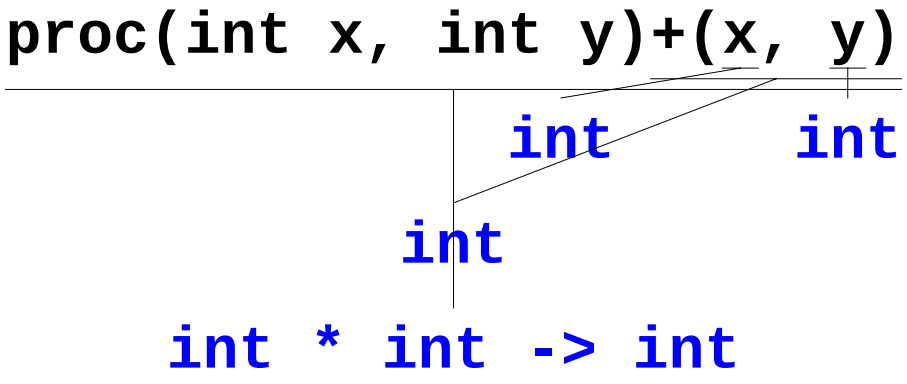
Types

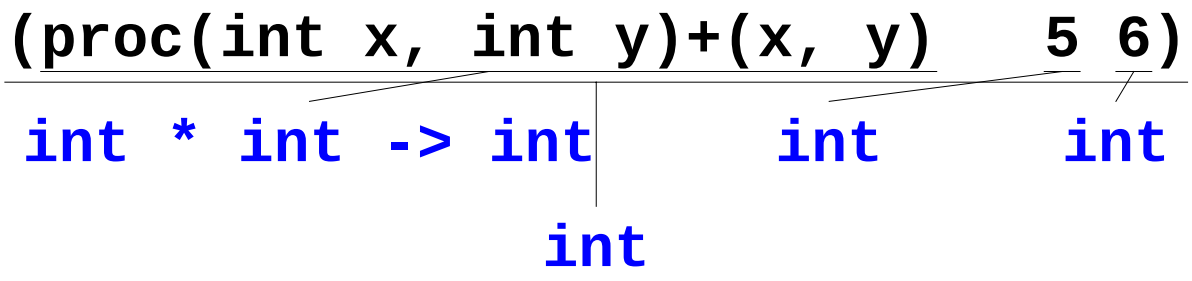

(proc(bool x)if x then 1 else 2 true)
bool -> int
bool
int

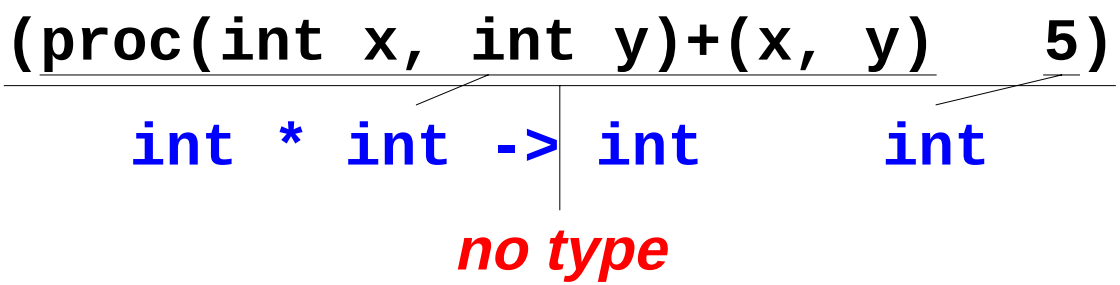

(proc(bool x)if x then 1 else 2 5)
bool -> int
int
no type


(7 5)
int
int
no type

Types


proc(int x, int y)+(x, y)
int
int
int
int * int -> int


(proc(int x, int y)+(x, y) 5 6)
int * int -> int
int
int
int


(proc(int x, int y)+(x, y) 5)
int * int -> int
int
no type

New Interpreter and Checker

- Change our interpreter:
  - Add types for arguments and letrec results to the grammar
- Implement a type-checker:
  - Recursively assign types to subexpressions
  - Check consistency at **`if`** and application
  - Treat primitives as built-in functions

```
+ : int * int -> int
```