



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- Starting call-by-value...

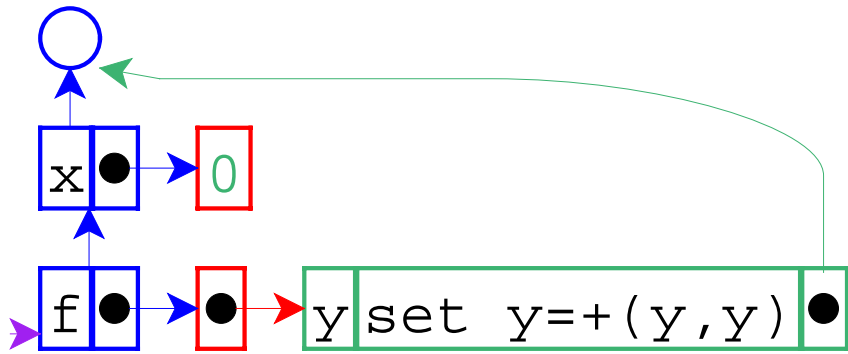


call-by-value

```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- Eval RHSs

call-by-value

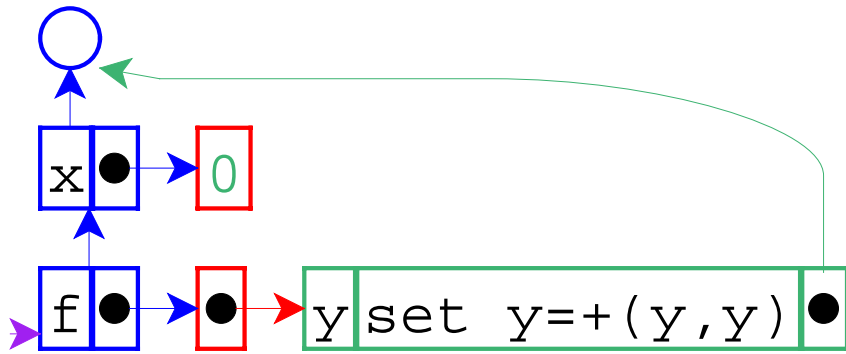


*technically, should be one frame with both x and f

```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- Bind x and f to 0 and closure, respectively

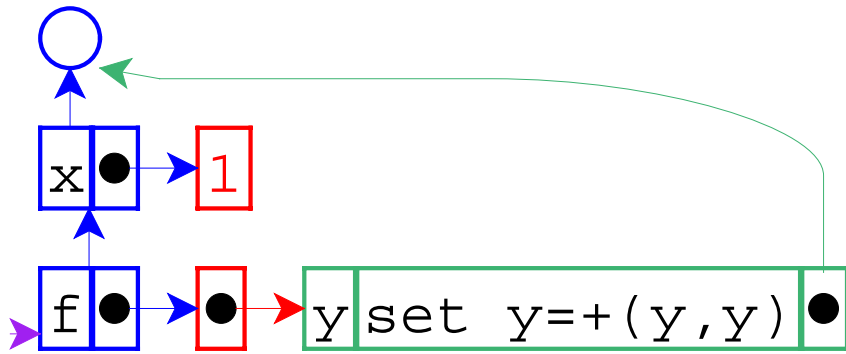
call-by-value



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- Eval RHS for z

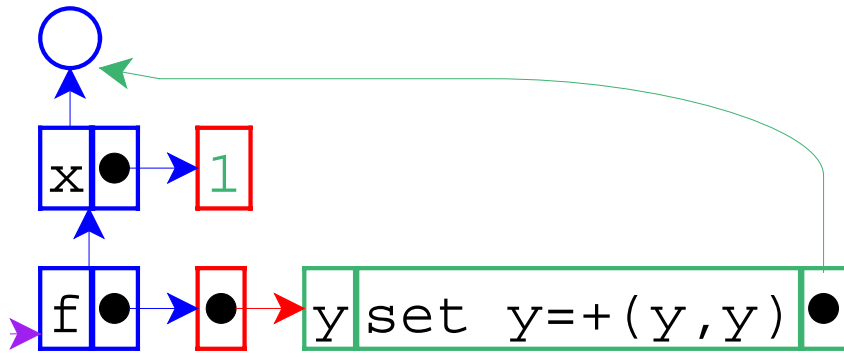
call-by-value



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Value for x changed to 1

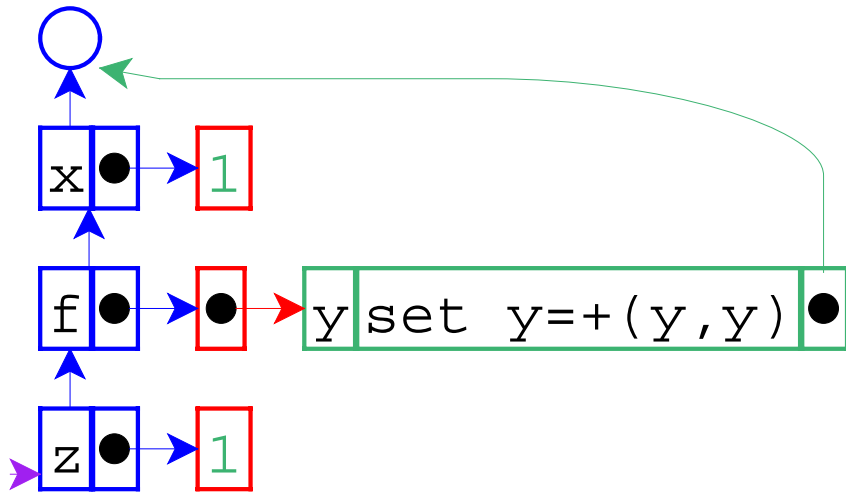
call-by-value



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Return x...

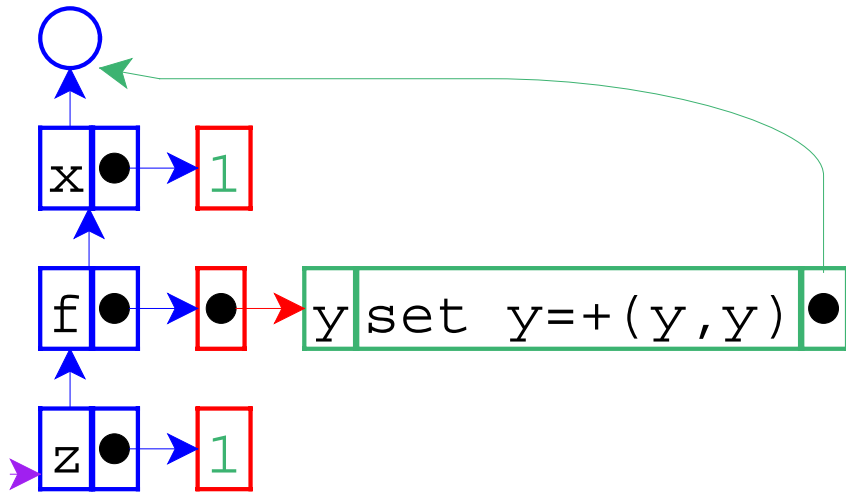
call-by-value



```
let x = 0
  f = proc(y) set y=+(y,y)
  in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- ... and bind `z` to the result, `1`

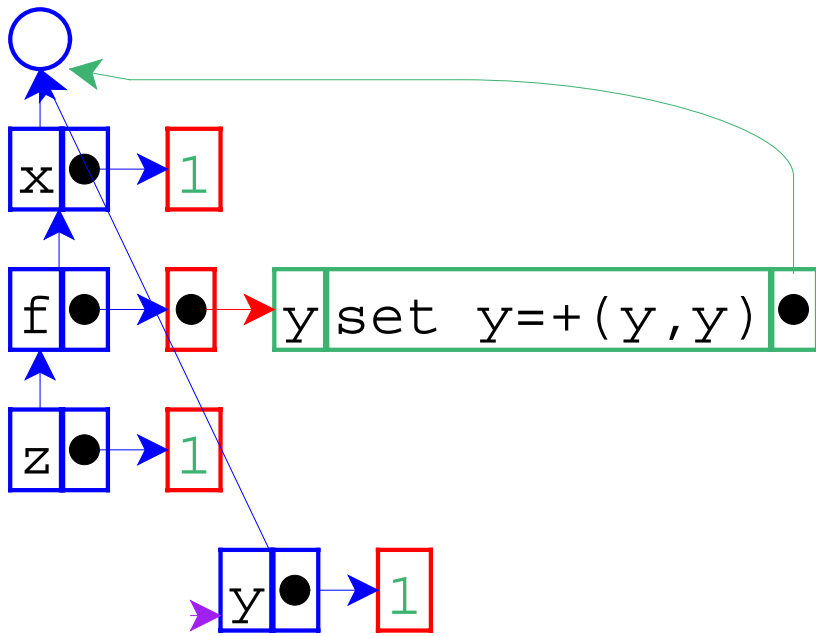
call-by-value



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- Call `f` with `z`

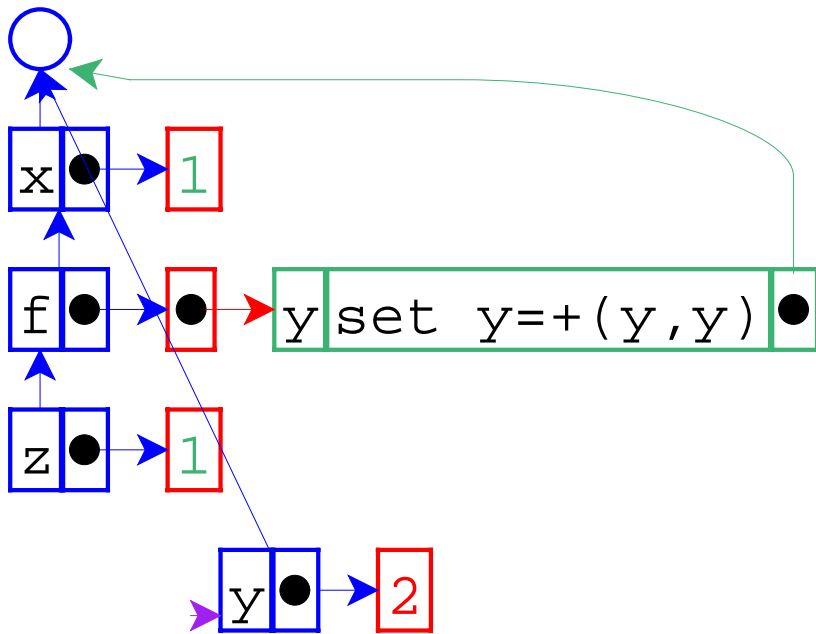
call-by-value



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Call-by-value creates a new location for `y`

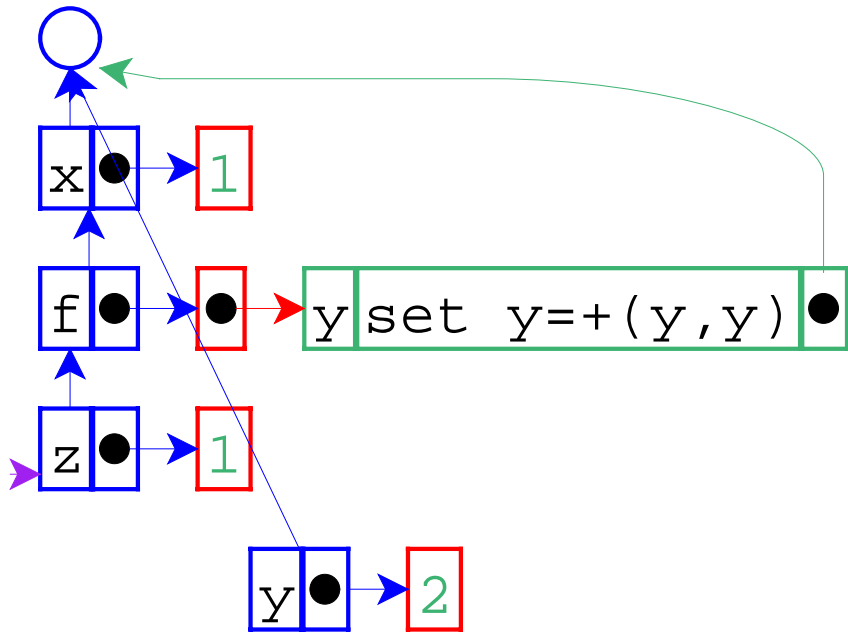
call-by-value



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Value for `y` changed to 2

call-by-value



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Result is the current value of `z`: 1



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

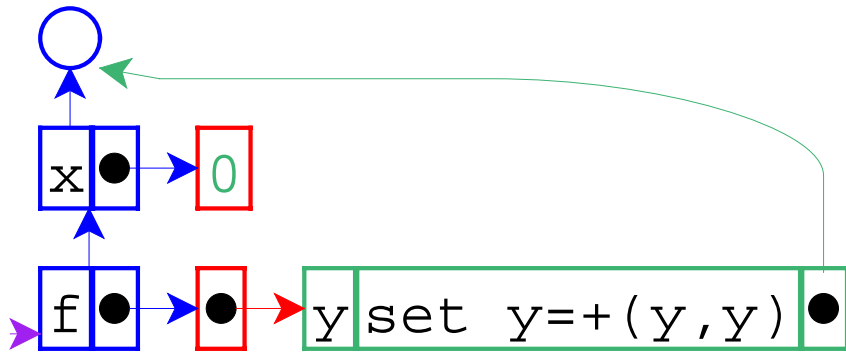
- Starting call-by-reference...



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- Eval RHSs

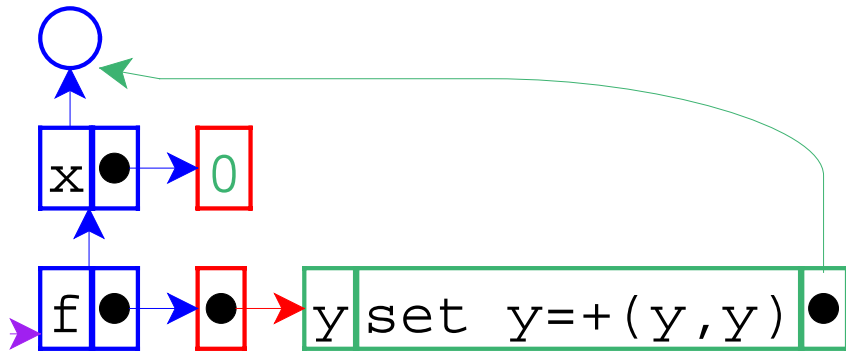
call-by-reference



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- Bind x and f to 0 and closure, respectively

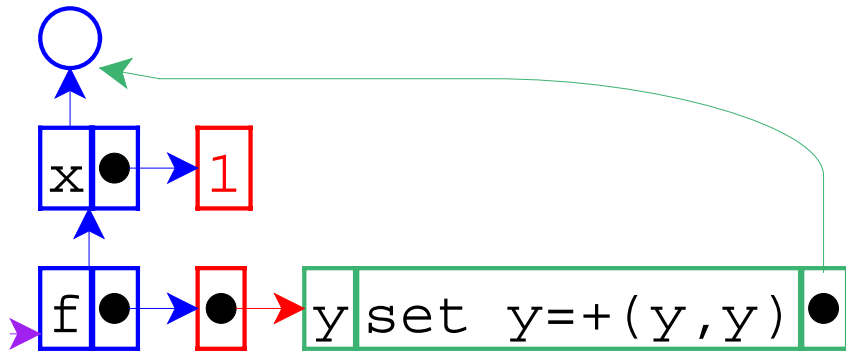
call-by-reference



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- Eval RHS for z

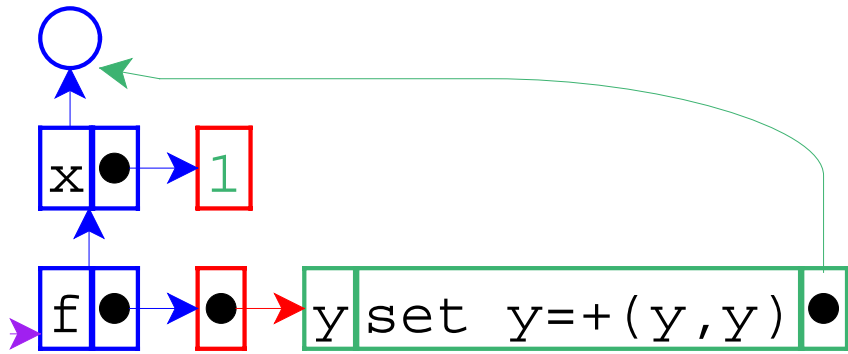
call-by-reference



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Value for x changed to 1

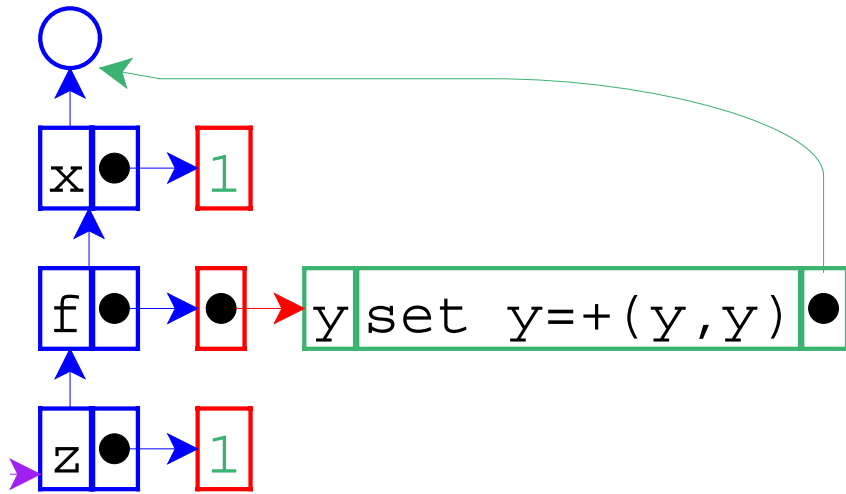
call-by-reference



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Return x...

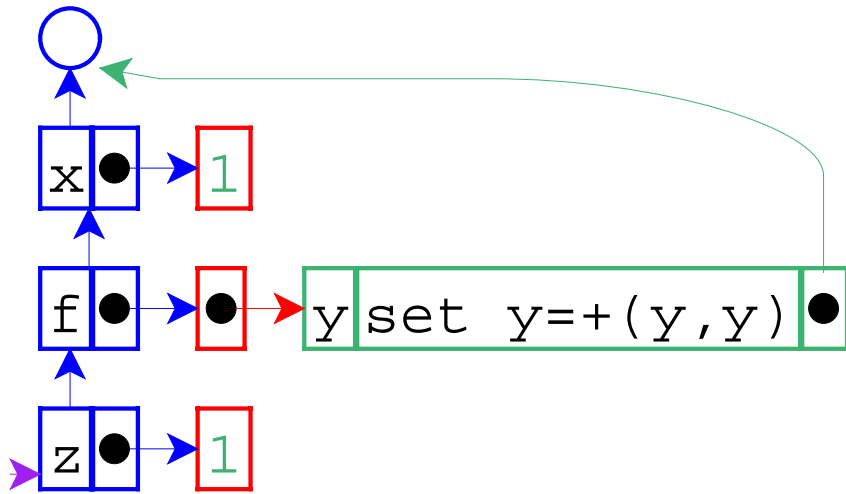
call-by-reference



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- ... and bind `z` to the result, 1

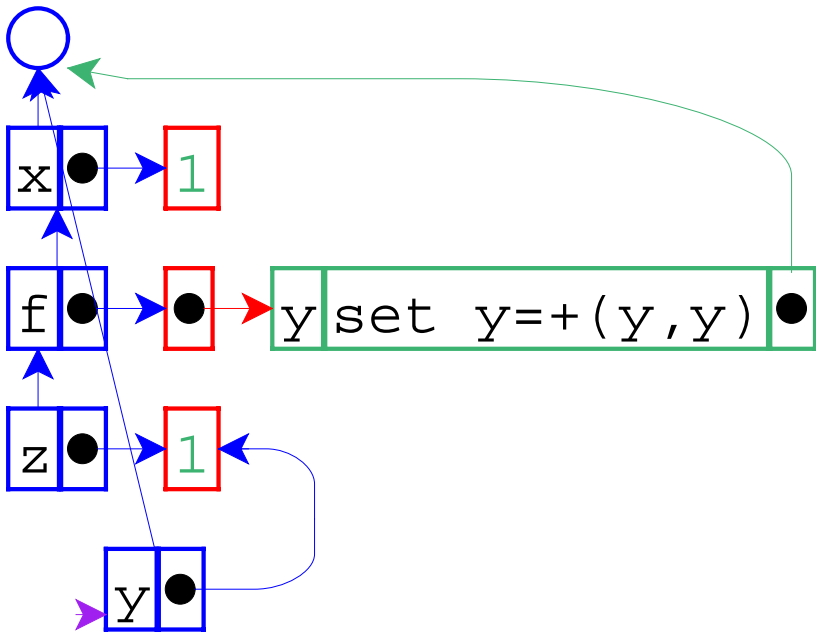
call-by-reference



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Call f with z

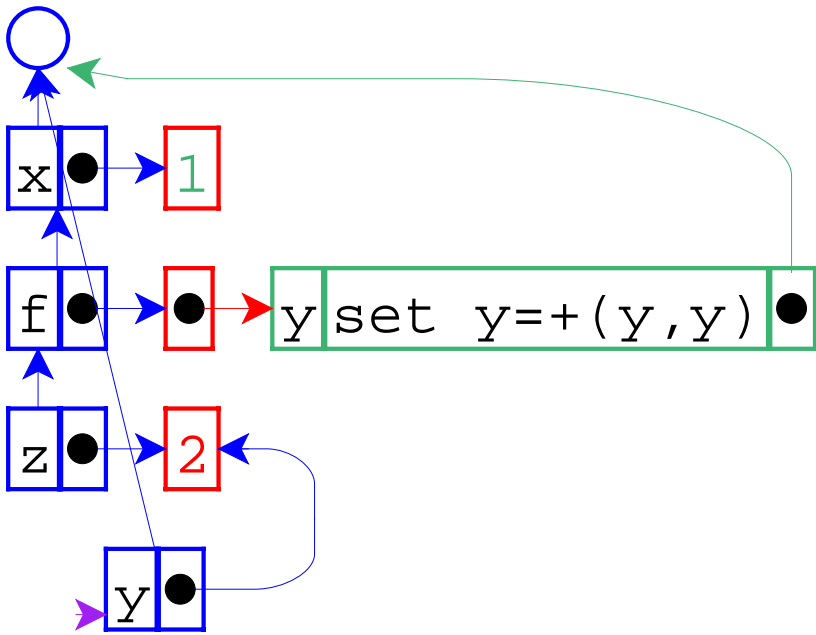
call-by-reference



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Call-by-reference shares location for z with y

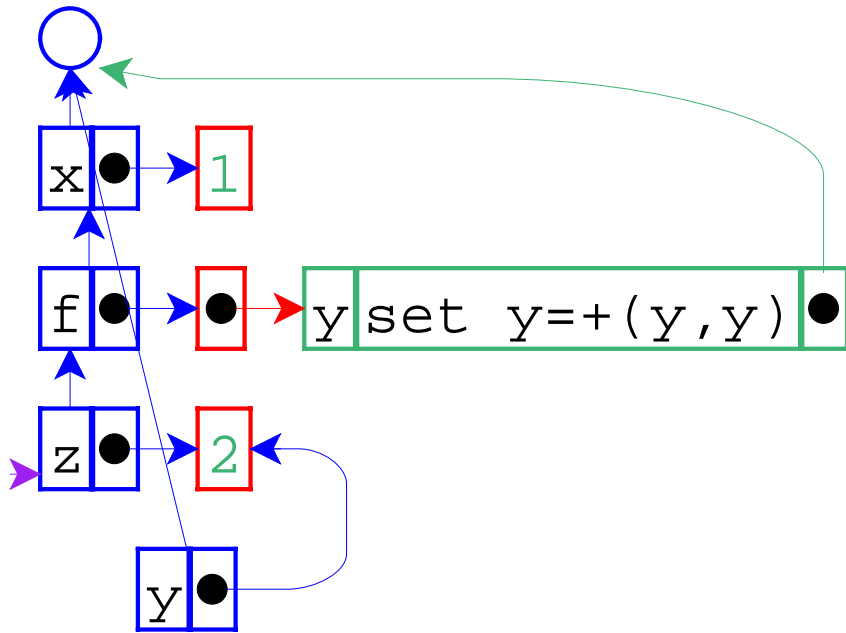
call-by-reference



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Value for y (and therefore z) changed to 2

call-by-reference



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Result is the current value of `z`: 2



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

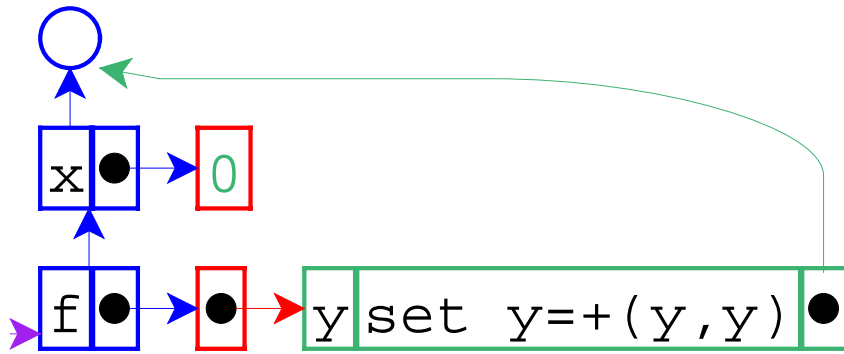
- Starting call-by-name...



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- Eval RHSs

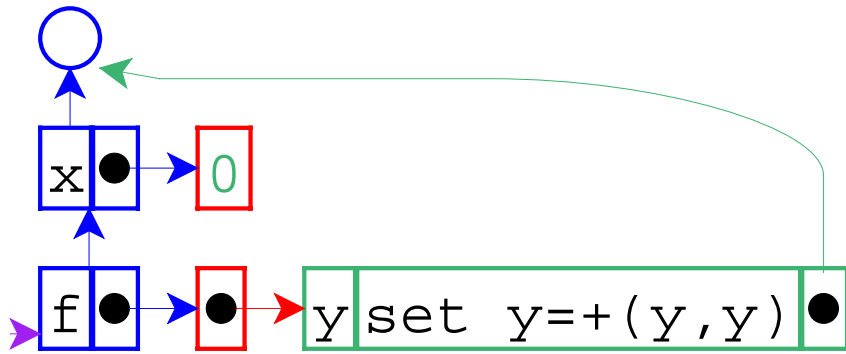
call-by-name



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- Simple expressions: bind x and f to 0 and closure, respectively

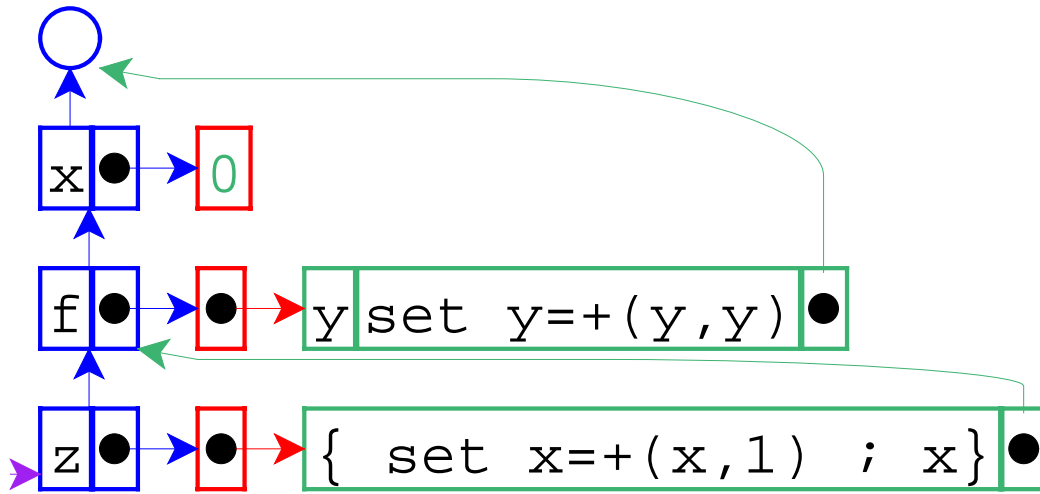
call-by-name



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- Handle RHS of z...

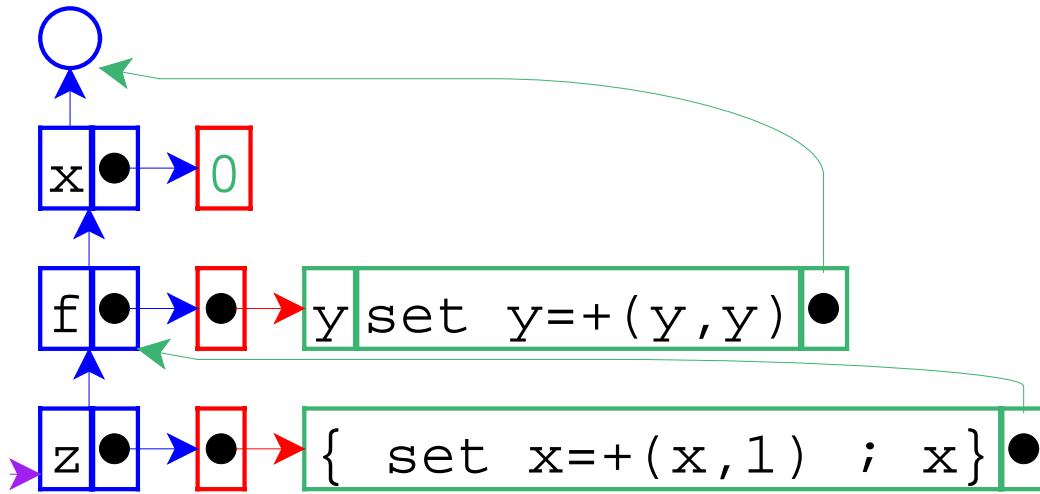
call-by-name



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- ... by creating a thunk for `z`

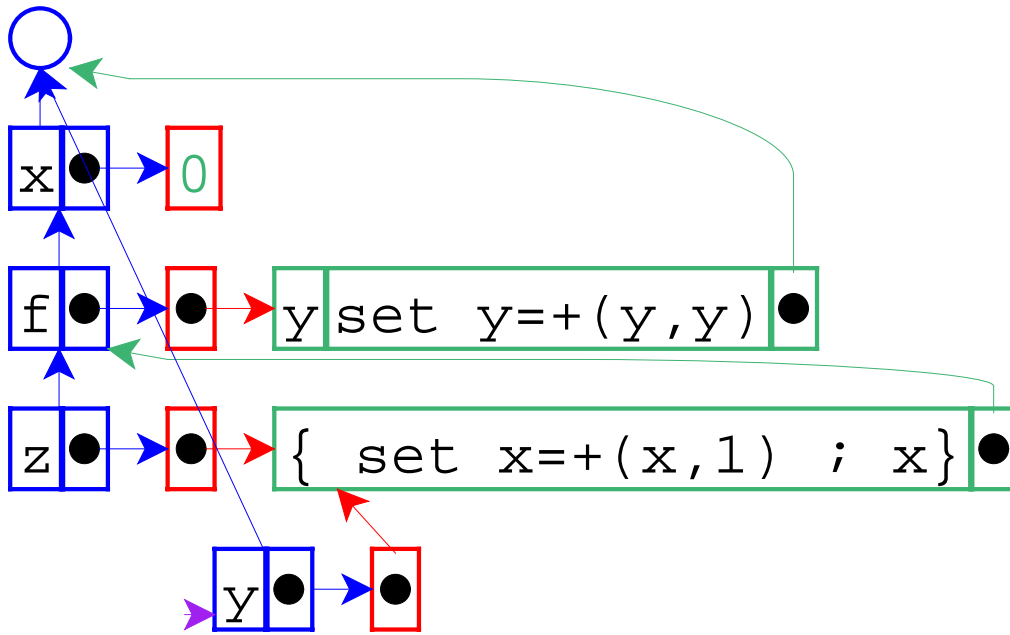
call-by-name



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Call f with z

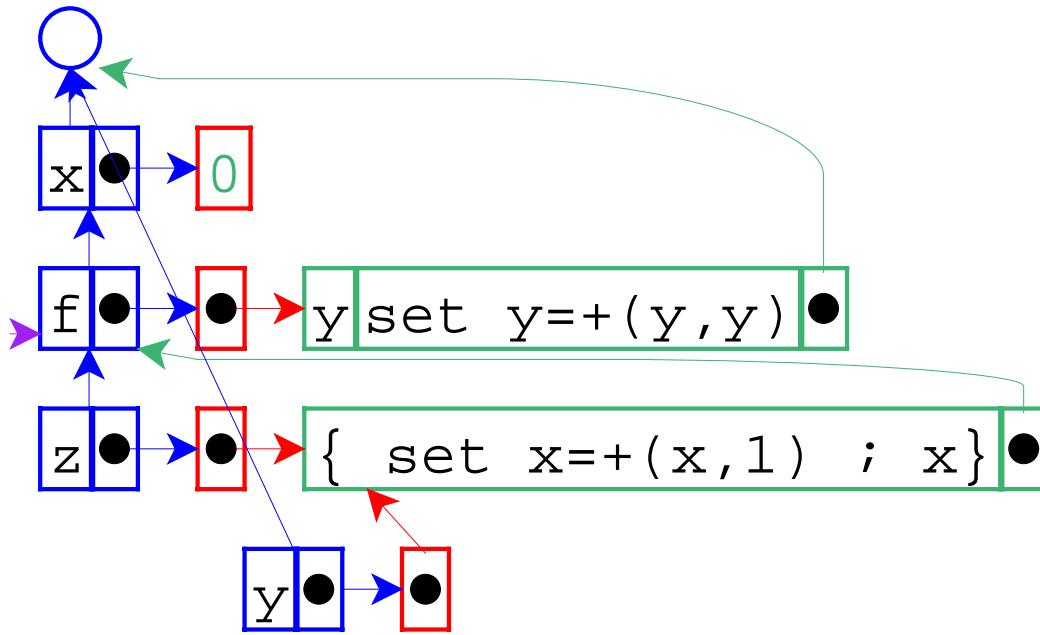
call-by-name



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Not by-reference; y gets a new location, containing the same thunk as z's location

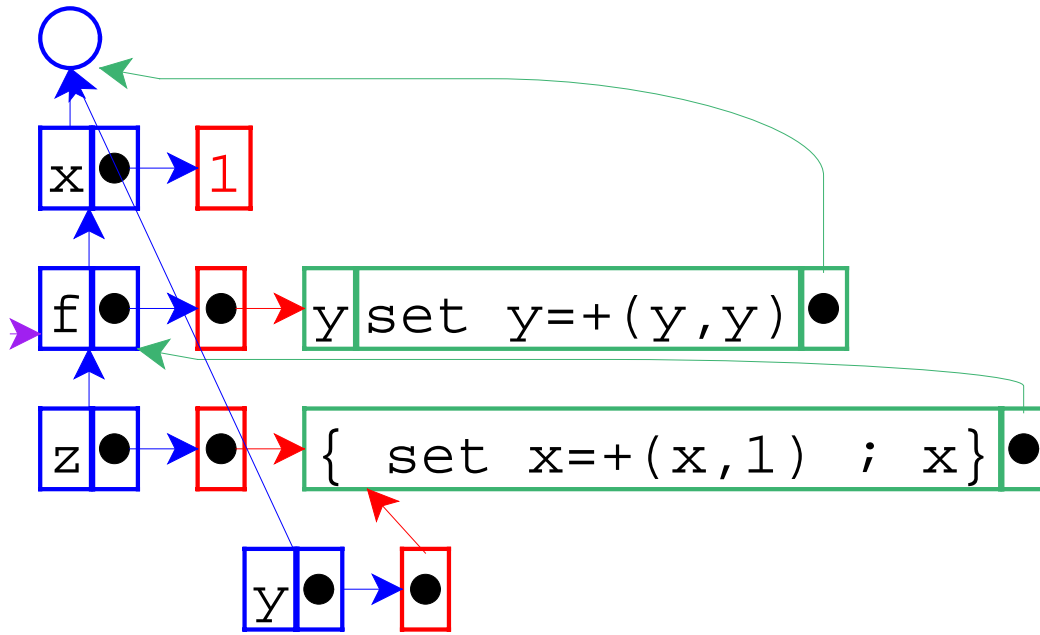
call-by-name



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- Use of `y` means we eval the thunk

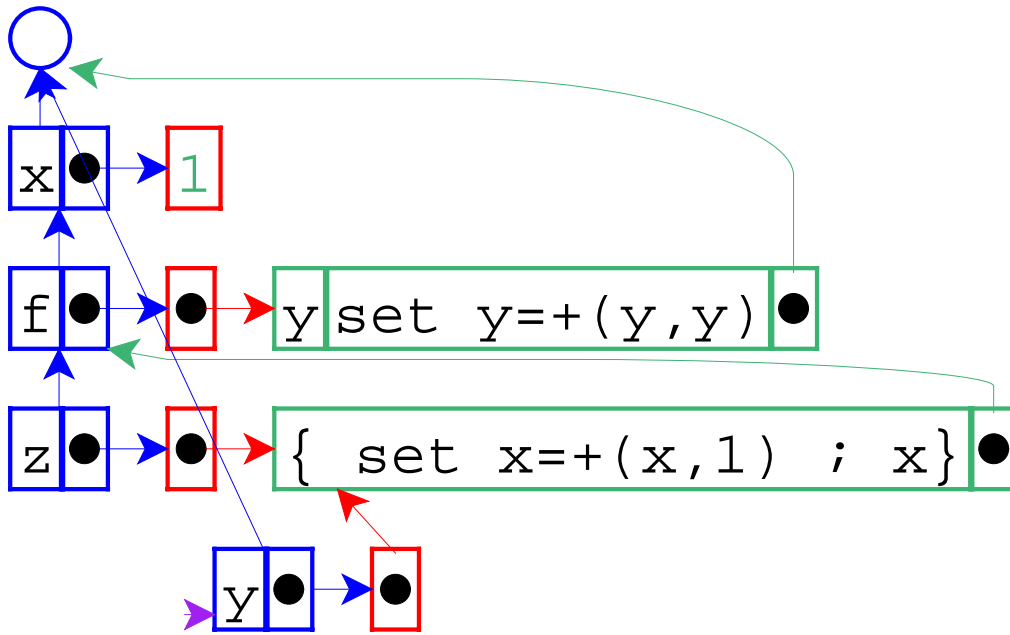
call-by-name



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Think changes value of x to 1

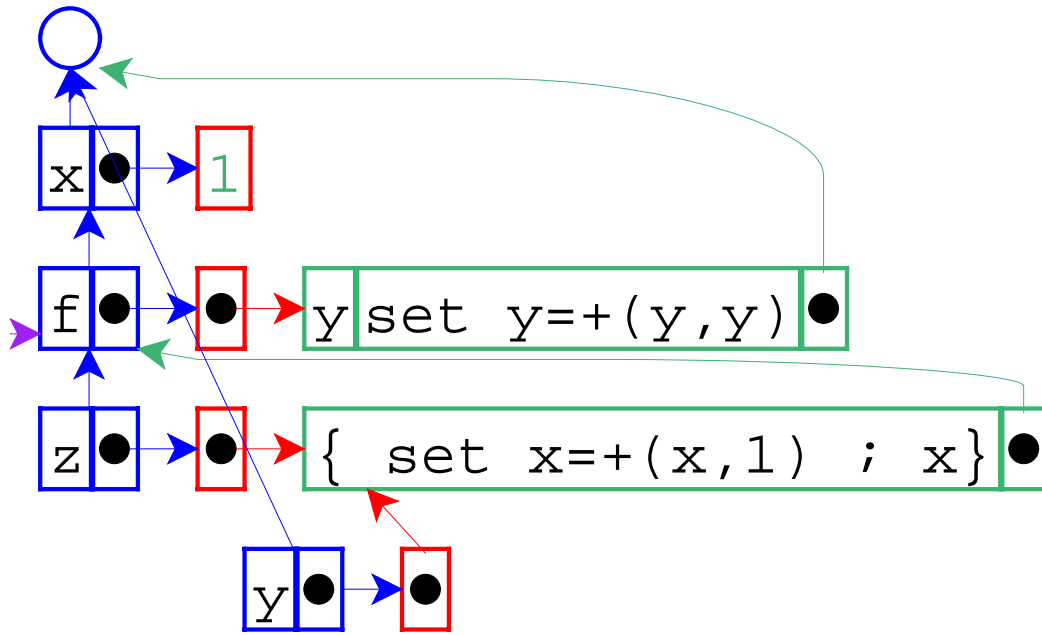
call-by-name



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Result for first use of y is 1

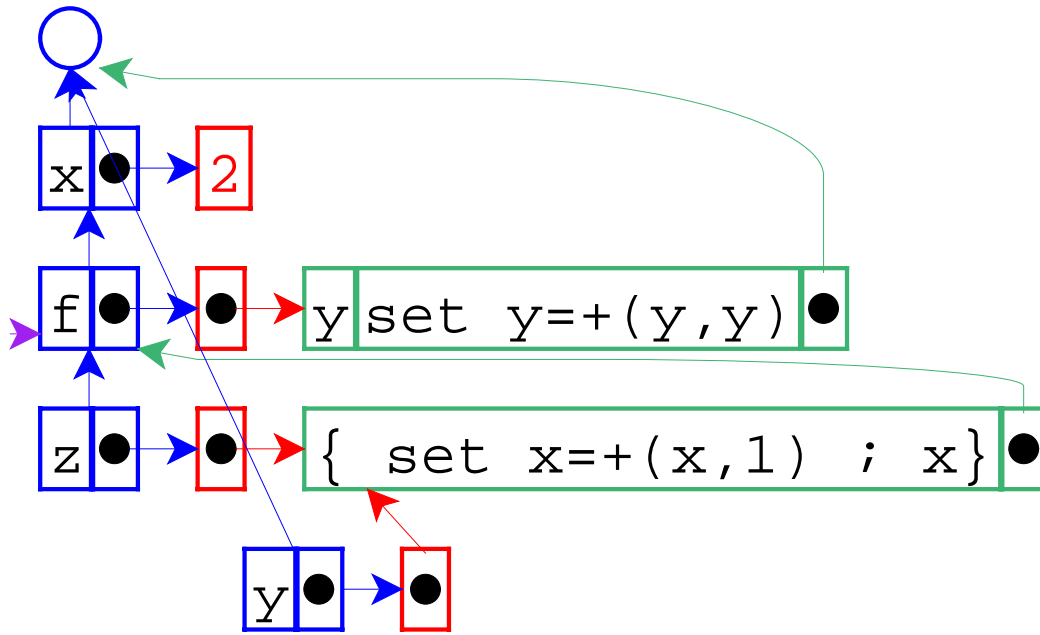
call-by-name



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- Another use of y means we eval the thunk again

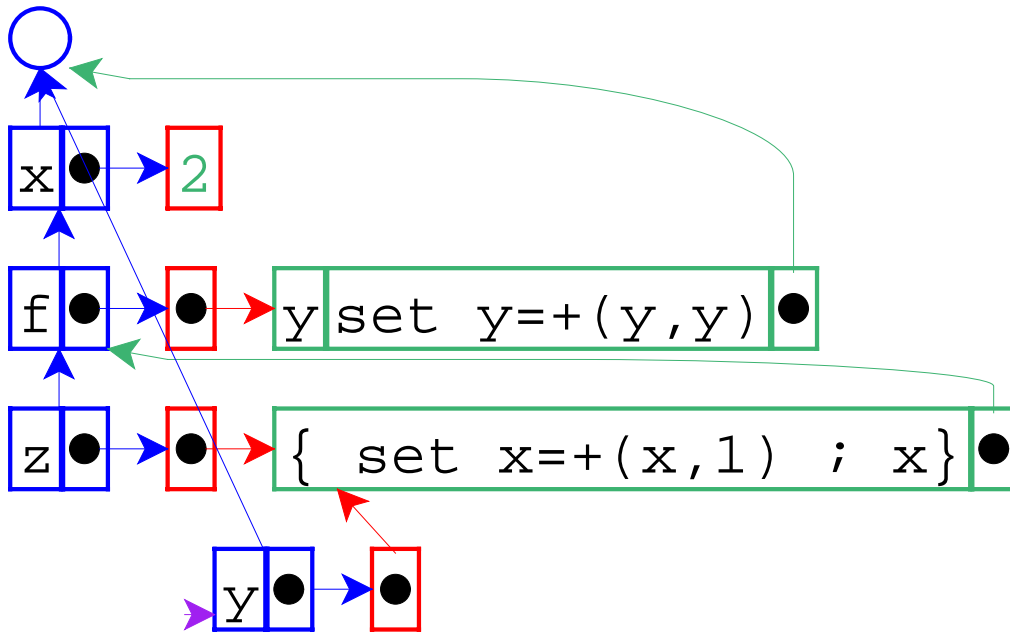
call-by-name



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Think changes value of x to 2

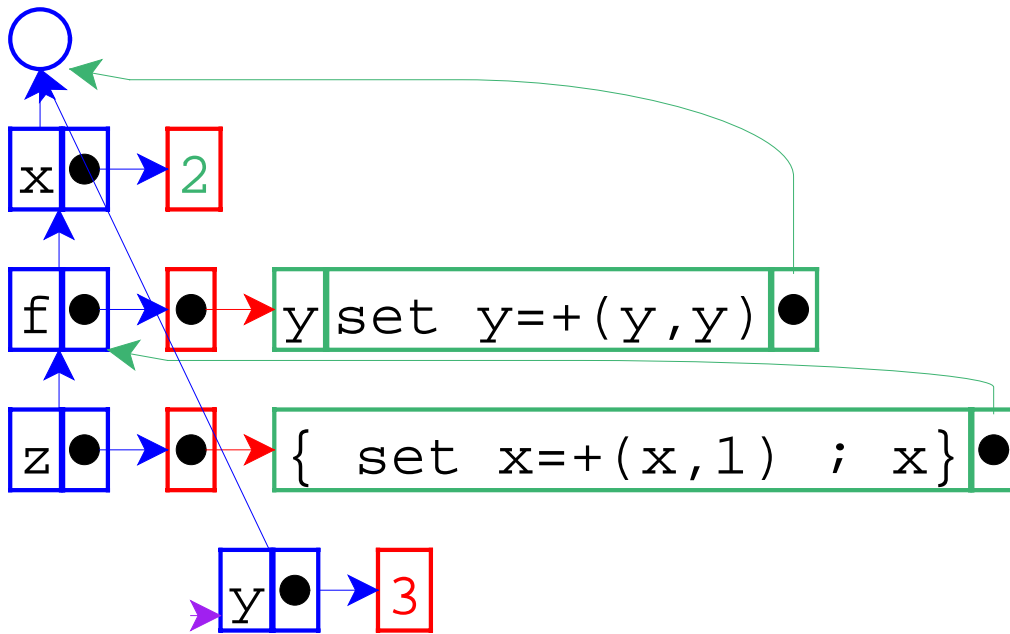
call-by-name



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Result for second use of y is 2

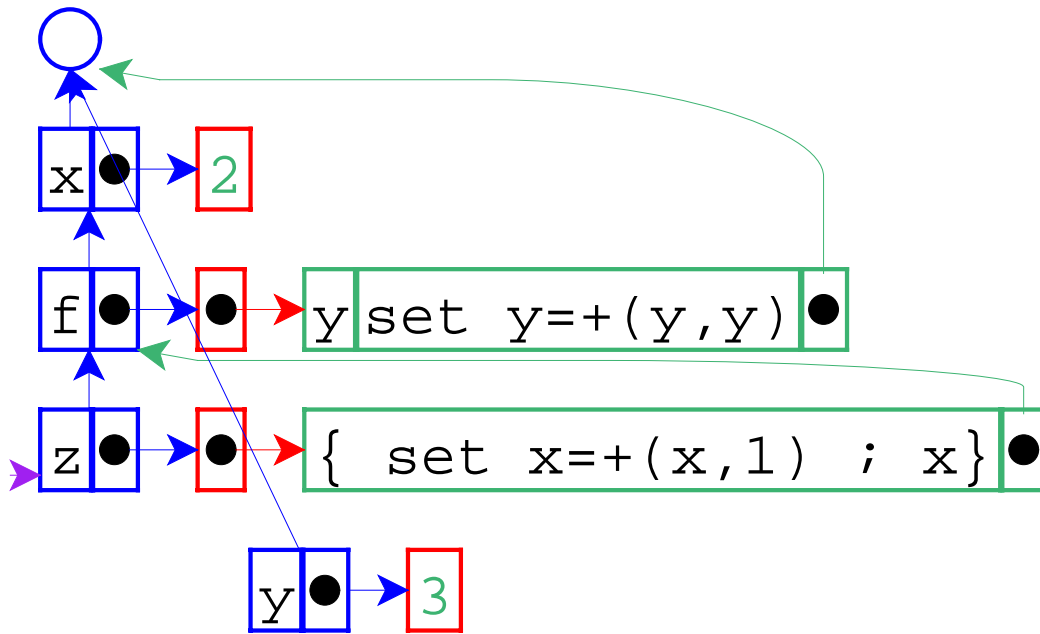
call-by-name



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Value for `y` changed to 3 (= 1 + 2)

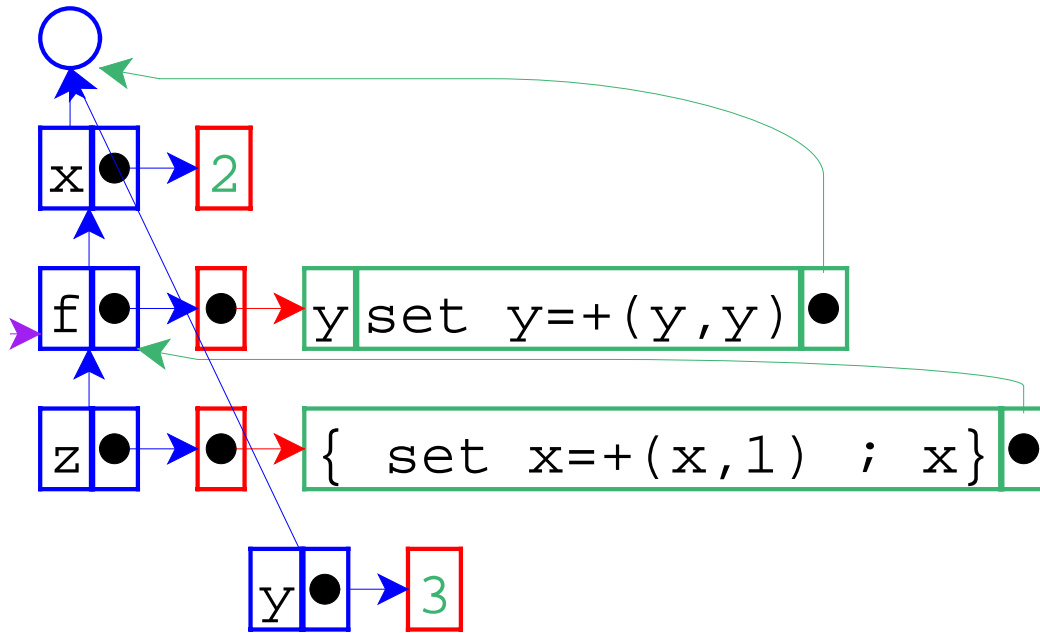
call-by-name



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Result is the value of `z`...

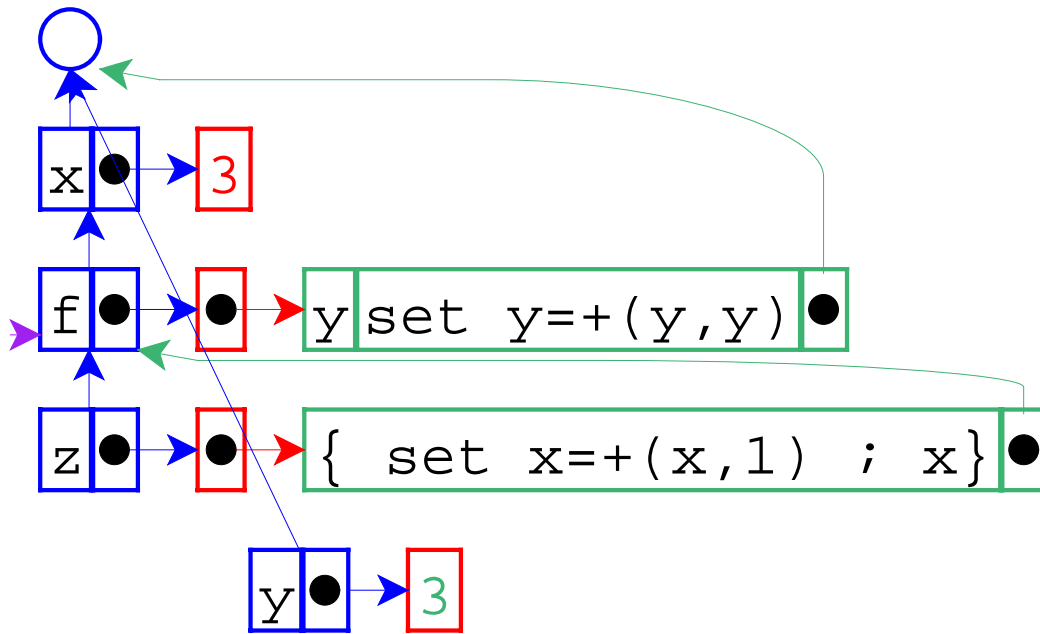
call-by-name



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- ... which means eval the thunk again

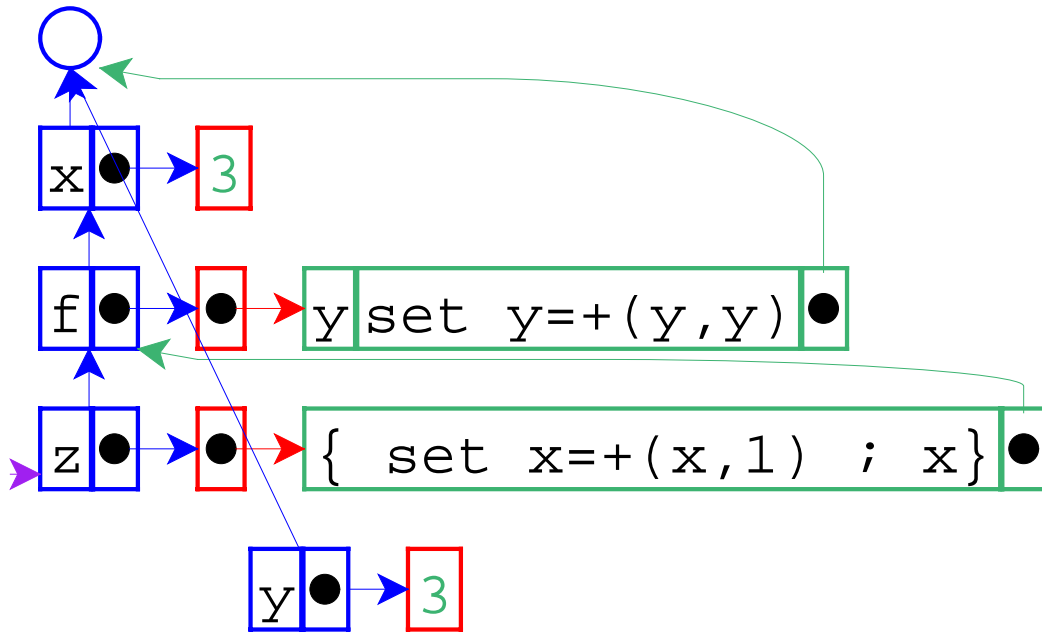
call-by-name



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Think changes value of `x` to 3

call-by-name



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- So 3 is the final result



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- Starting call-by-need...

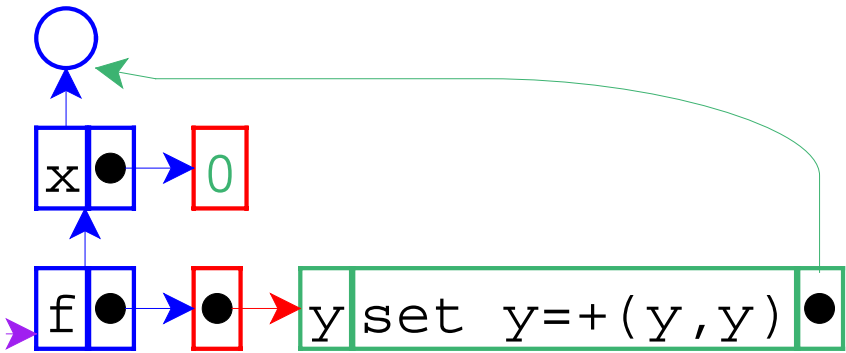


call-by-need

```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- Eval RHSs

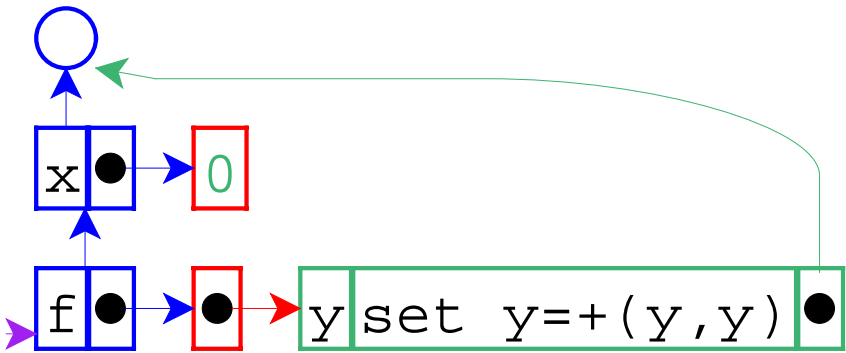
call-by-need



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- Simple expressions: bind `x` and `f` to `0` and closure, respectively

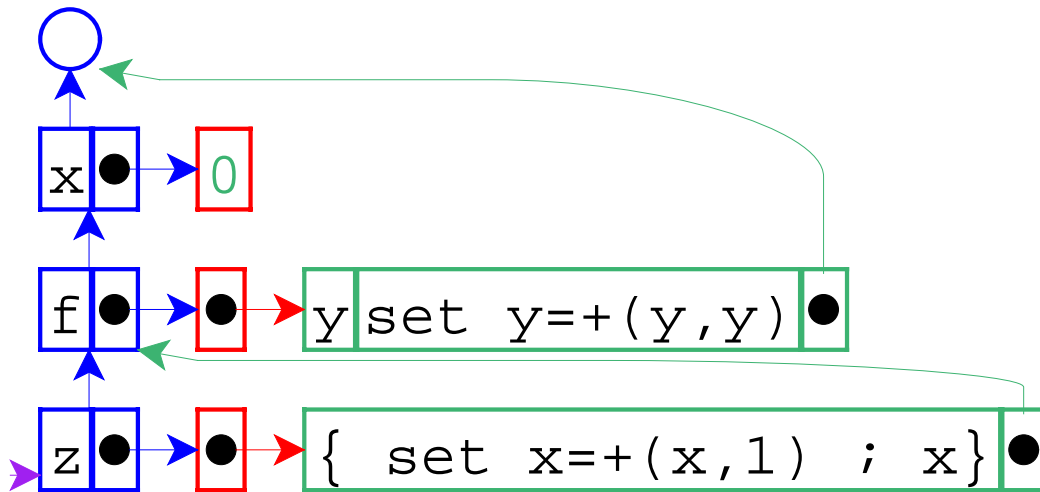
call-by-need



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- Handle RHS of z...

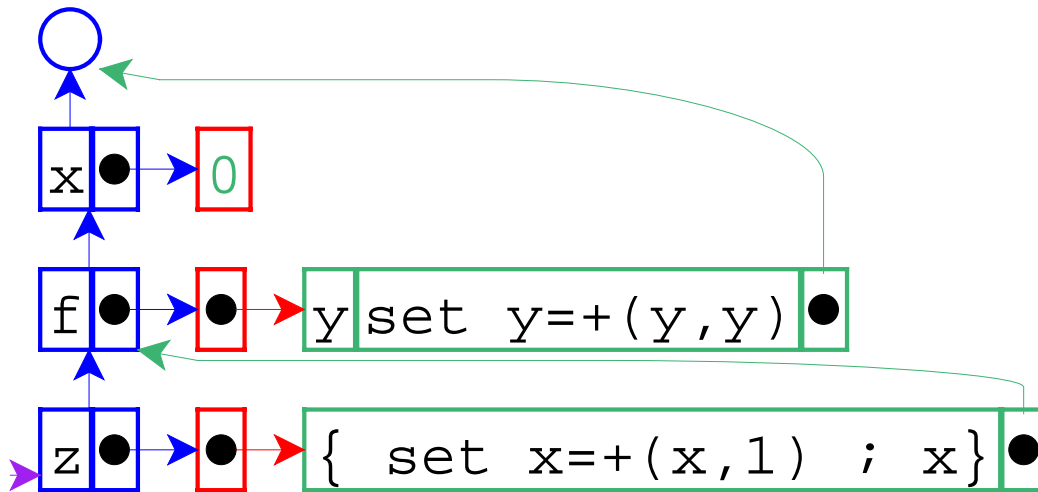
call-by-need



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- ... by creating a thunk for `z`

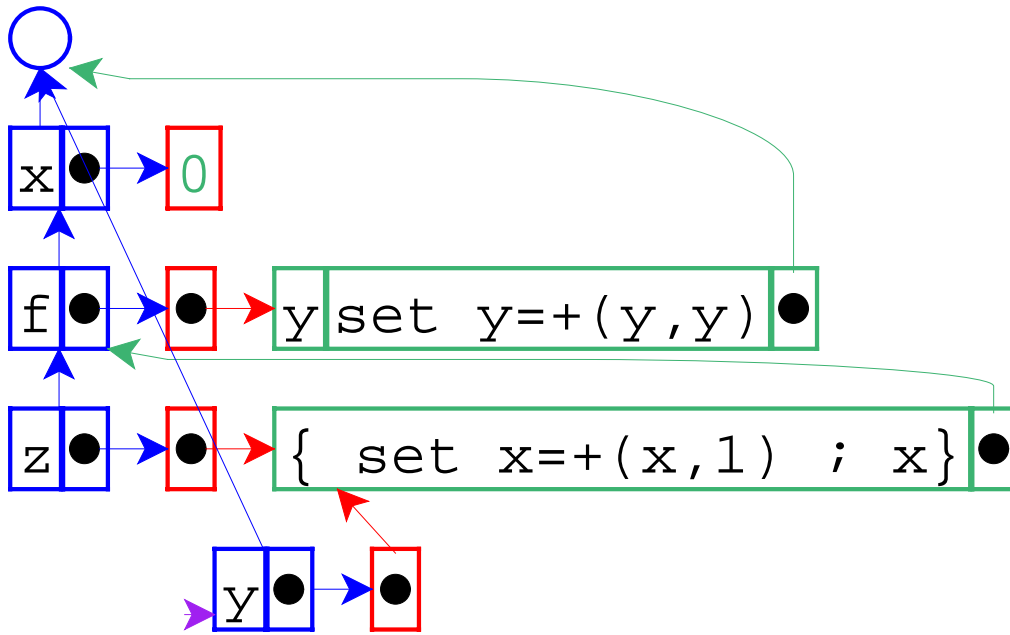
call-by-need



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- Call f with z

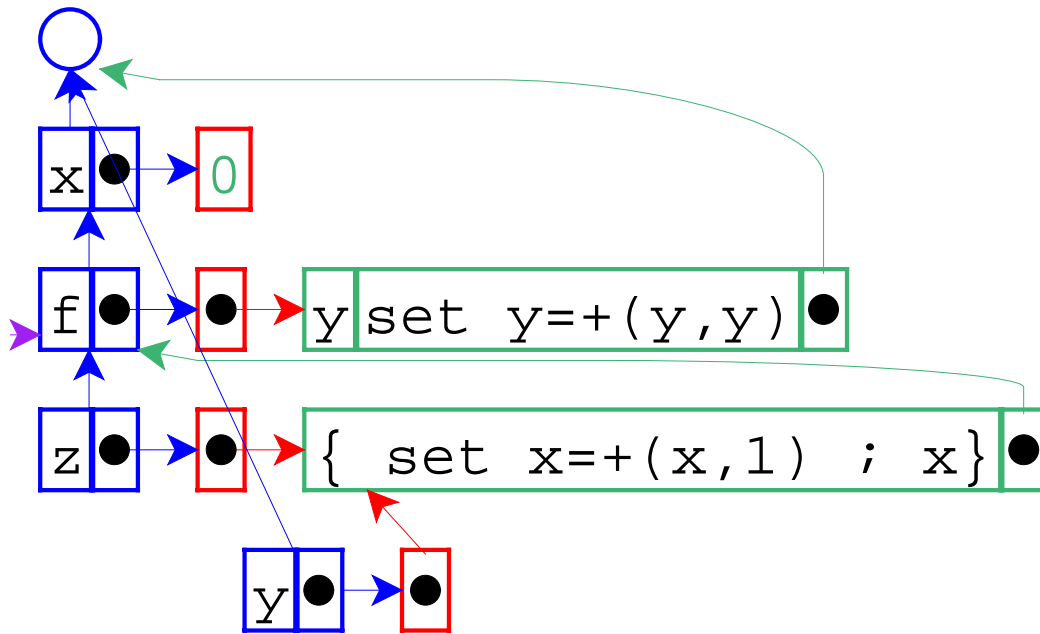
call-by-need



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- Not by-reference; y gets a new location, containing the same thunk as z's location

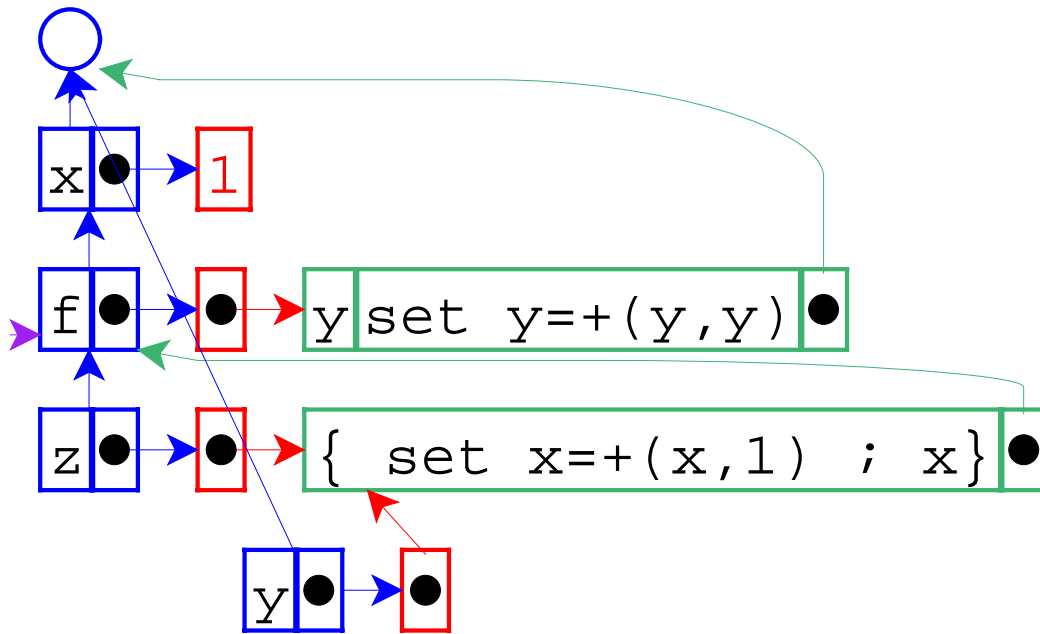
call-by-need



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- Use of y means we eval the thunk

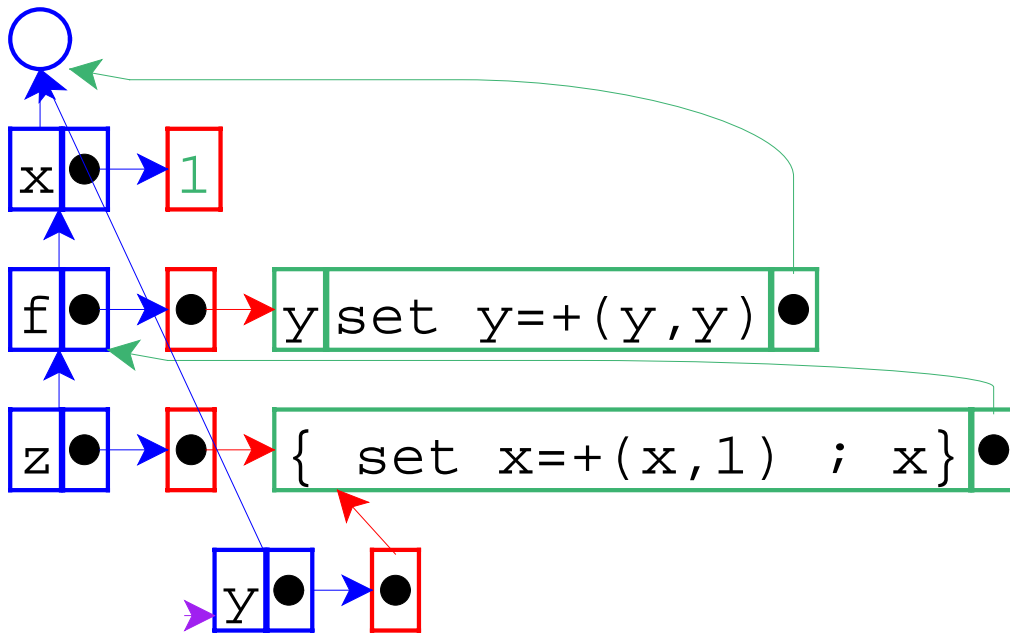
call-by-need



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- Think changes value of `x` to 2

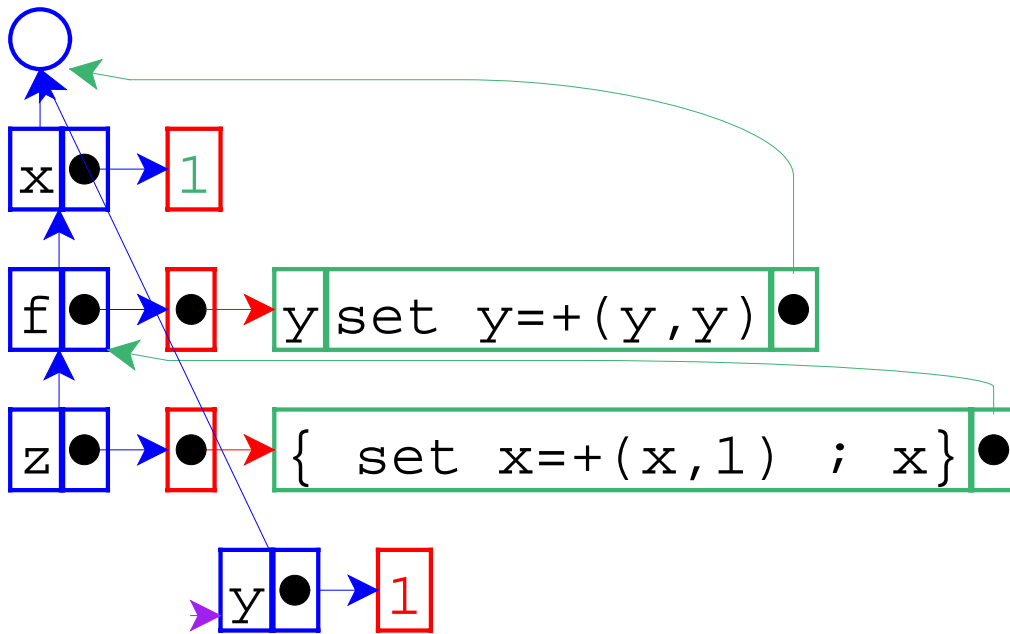
call-by-need



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- Result from first use of `y` was 1

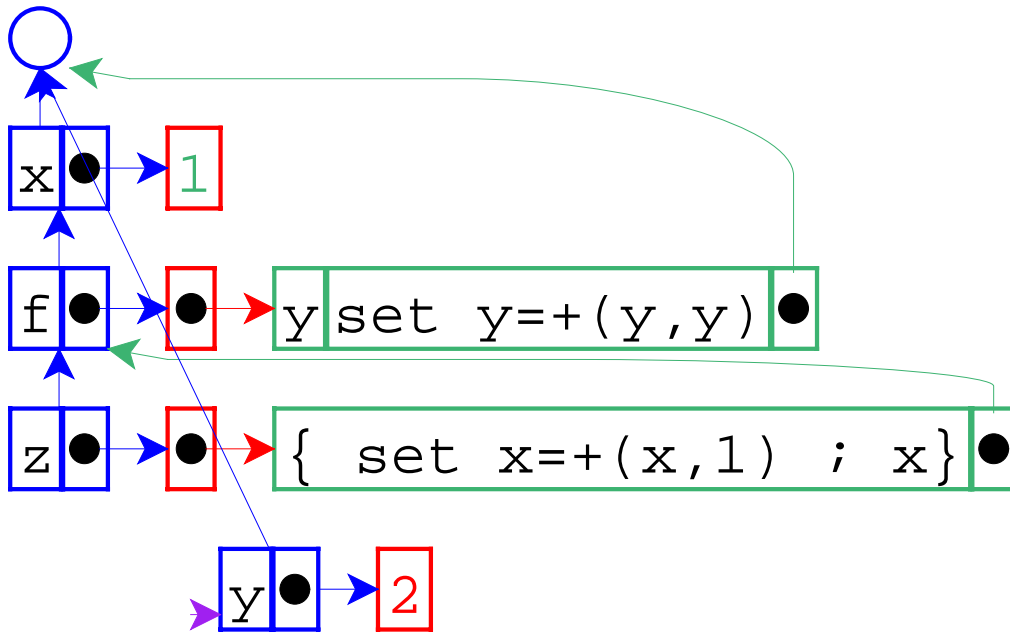
call-by-need



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- Since this is call-by-need, install the 1 into y

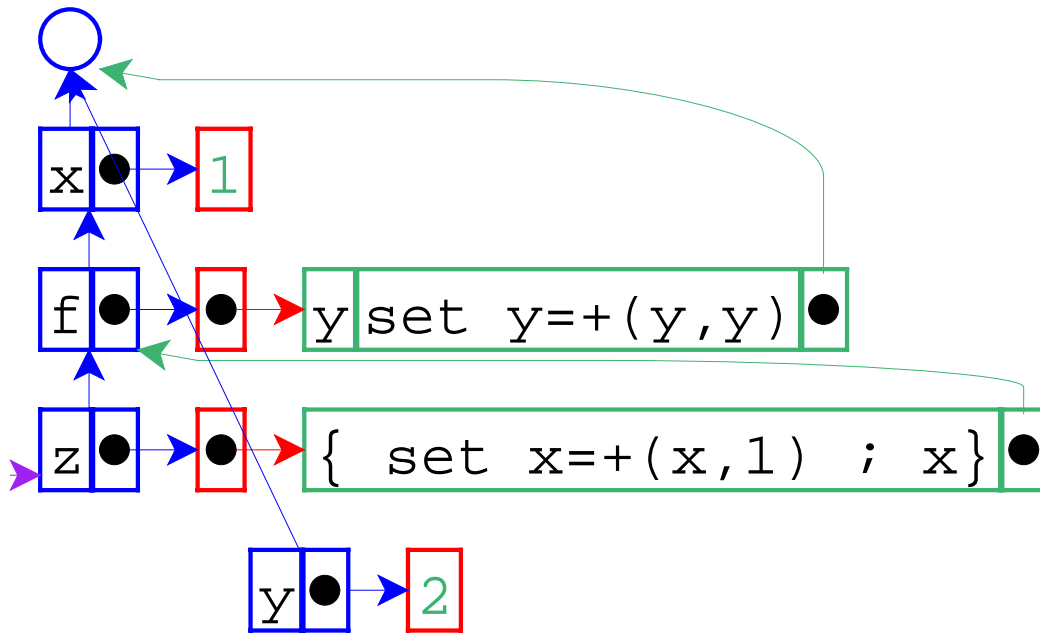
call-by-need



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Second use of y gets 1, set y to 2 (= 1+1)

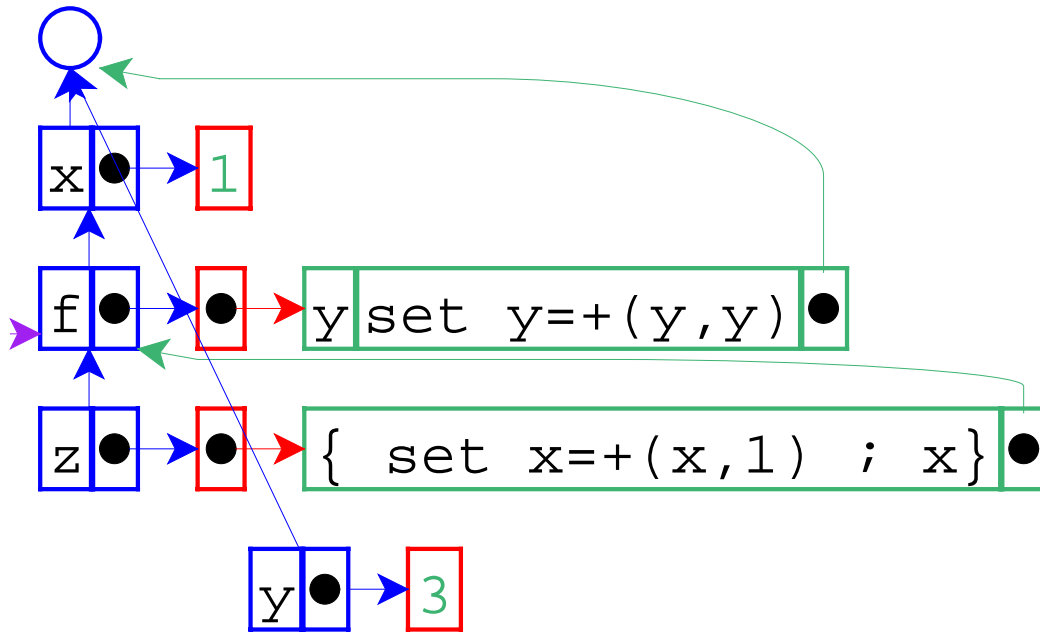
call-by-need



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- Result is value of z...

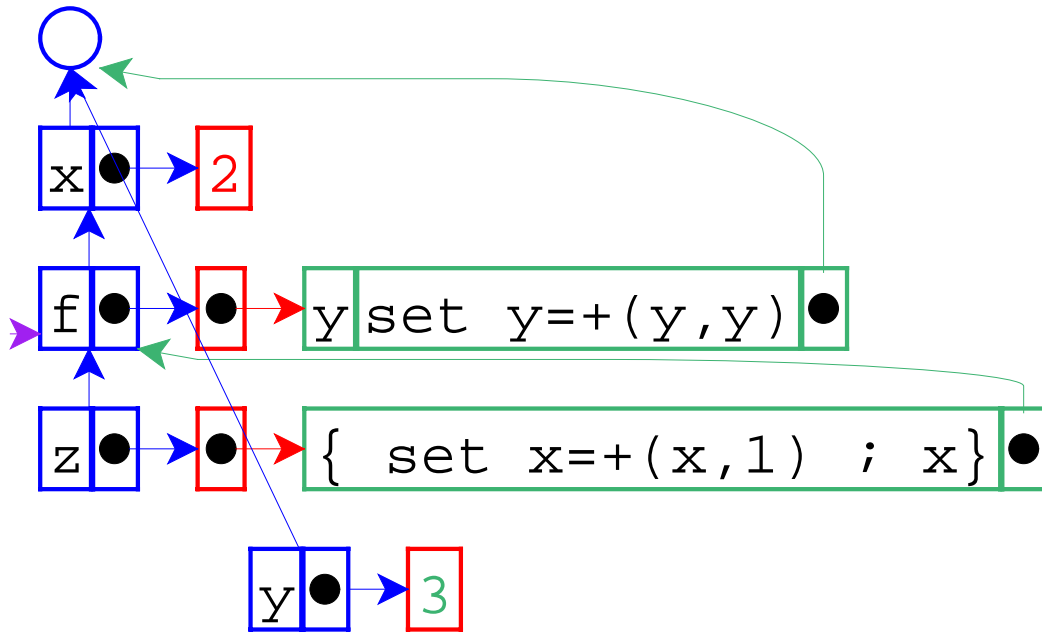
call-by-need



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- ... which means eval the thunk again (see note at end of this section)

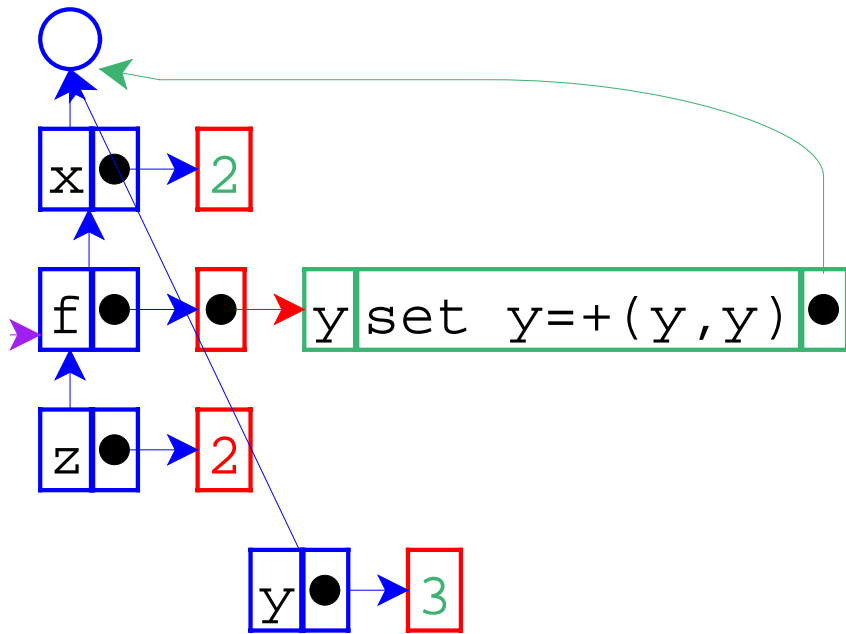
call-by-need



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Think changes value of `x` to 2

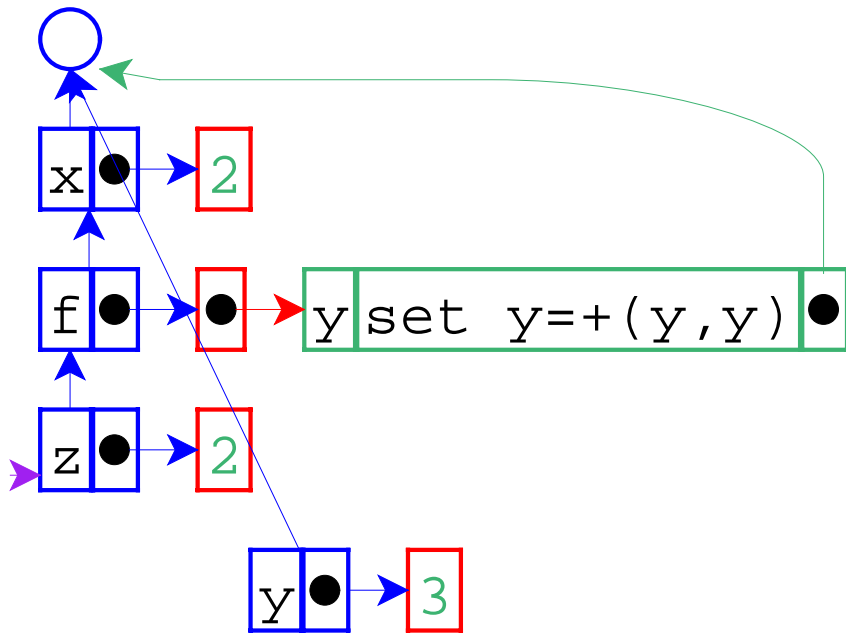
call-by-need



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- Result of thunk is 2; install result into z

call-by-need



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- Final result is from z: 2

Note:

- Our interpreter implements a strange kind of call-by-need, where using a variable in a function call can cause a thunk to be evaluated multiple times.
- This strangeness is an artifact of supporting call-by-reference, where we always treat variable arguments specially (even in the call-by-value case).



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

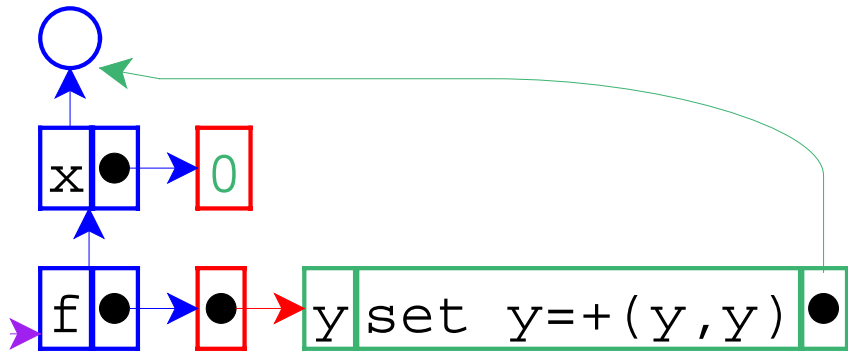
- Starting call-by-name combined with call-by-reference...



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- Eval RHSs

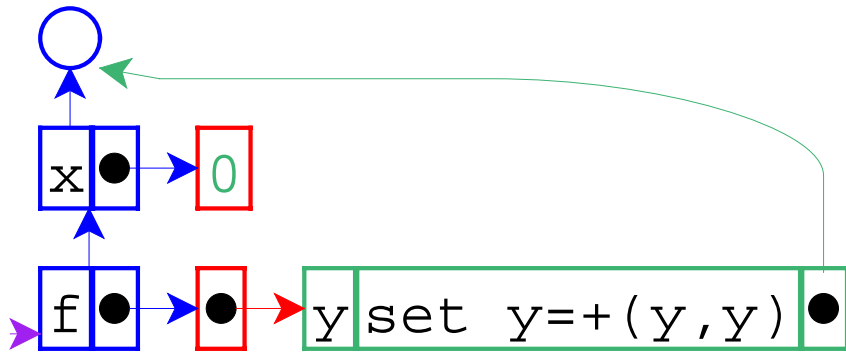
call-by-name/ref



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- Simple expressions: bind `x` and `f` to `0` and closure, respectively

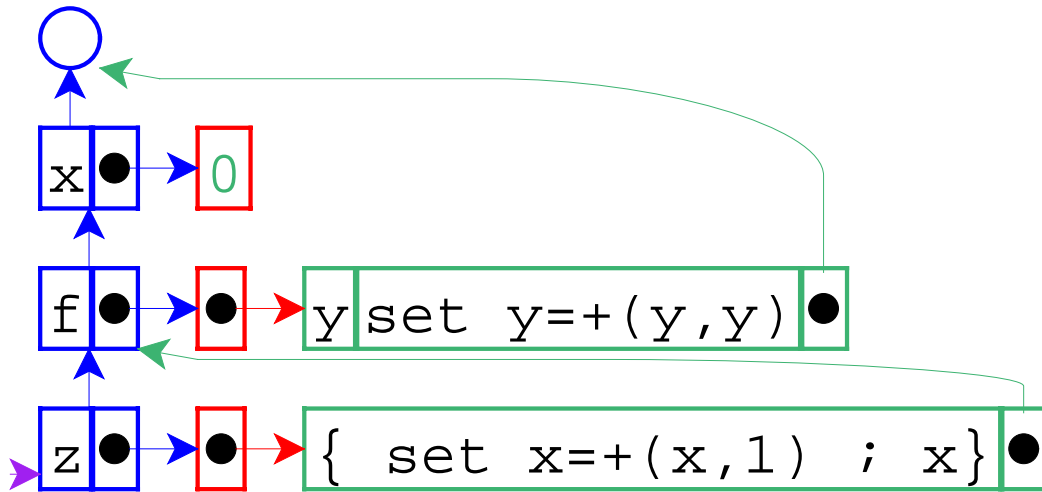
call-by-name/ref



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- Handle RHS of z...

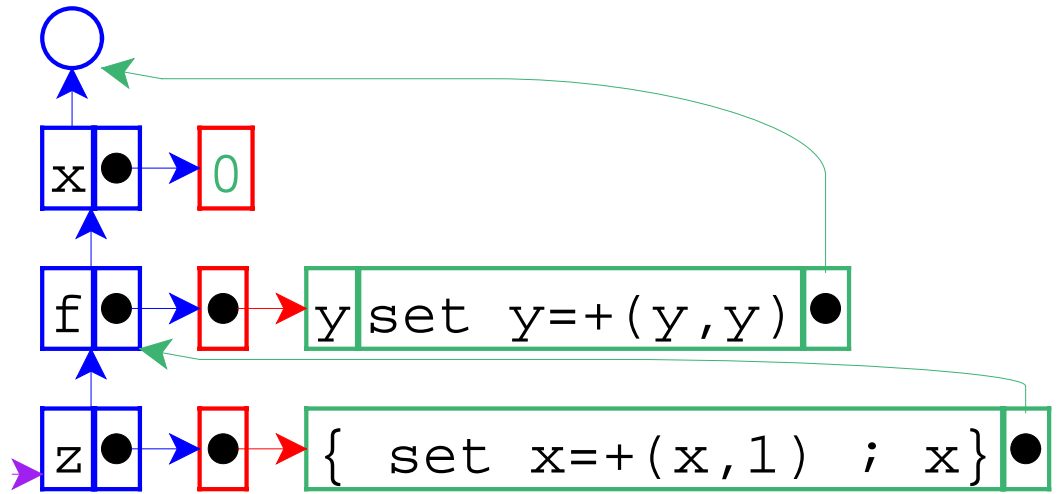
call-by-name/ref



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- ... by creating a thunk for `z`

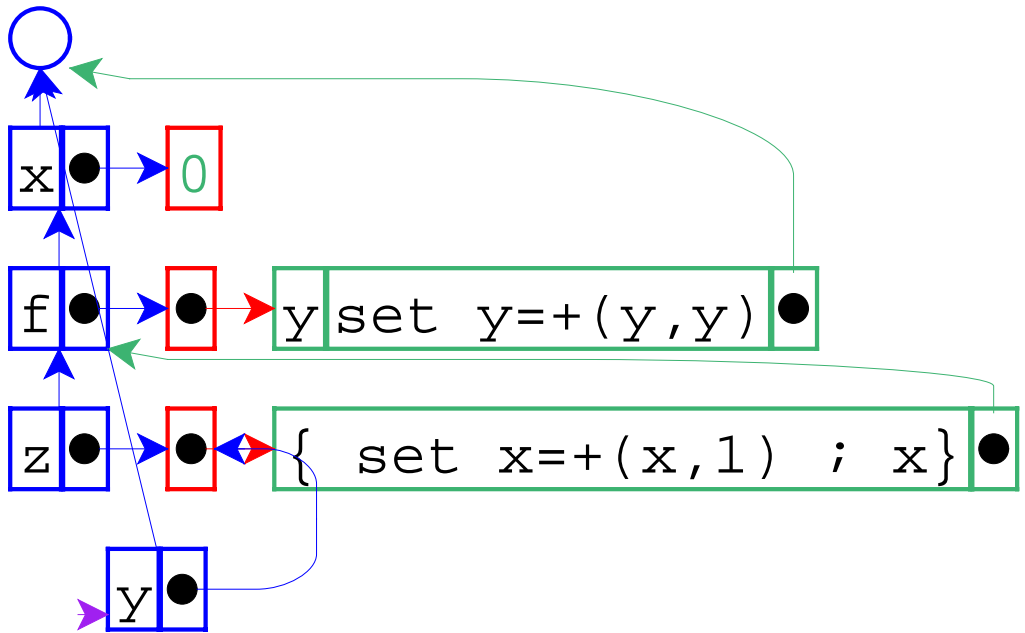
call-by-name/ref



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Call f with z

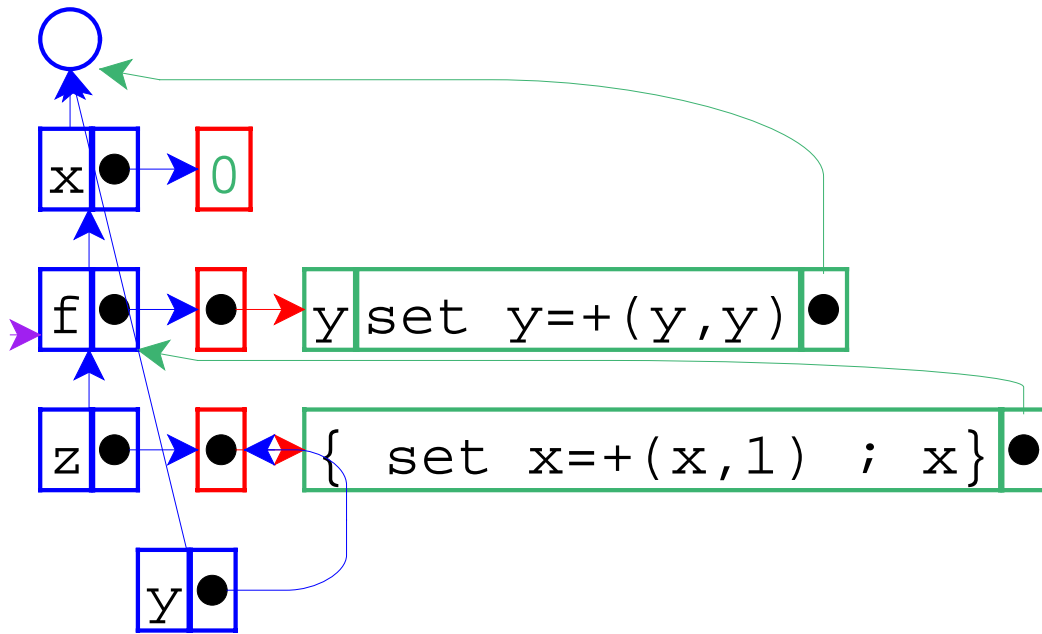
call-by-name/ref



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Call-by-reference shares location for z with y

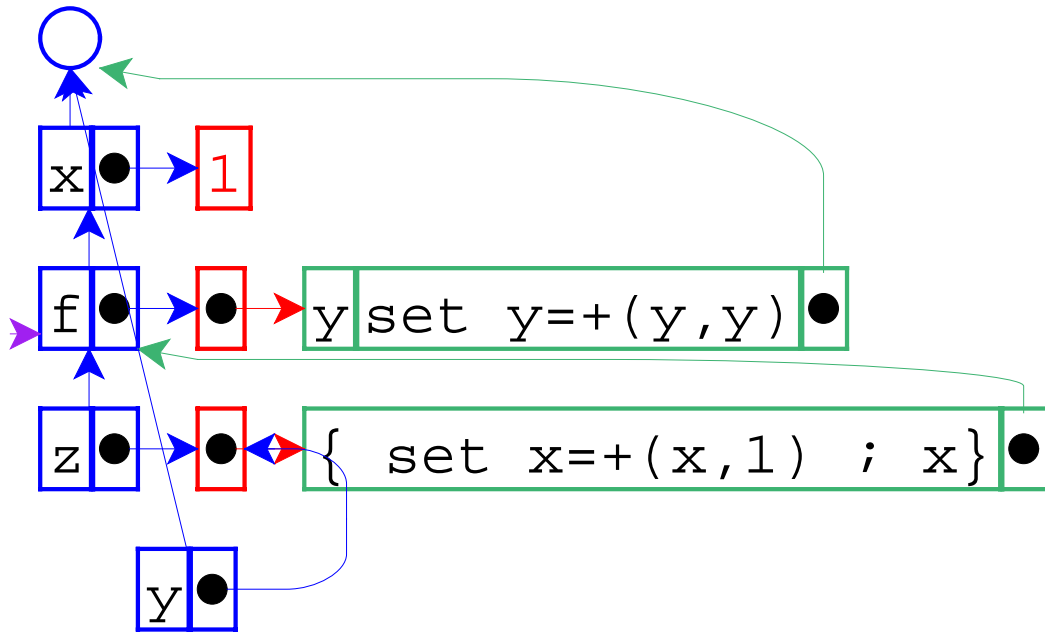
call-by-name/ref



```
let x = 0
    f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
    in { (f z) ; z }
```

- First use of `y` triggers evaluation of the thunk

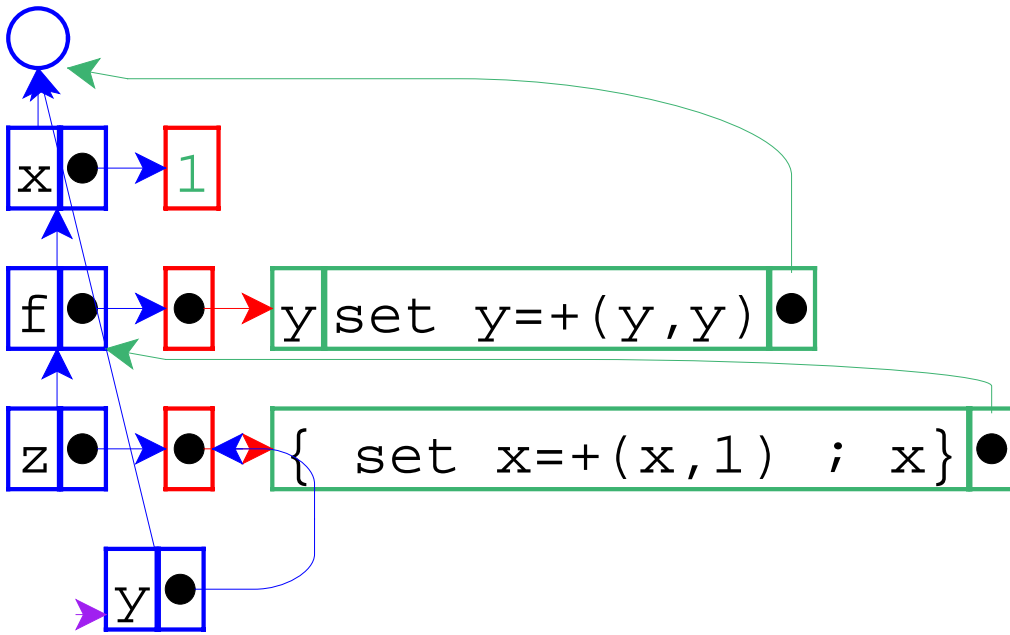
call-by-name/ref



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Think changes value of `x` to `1`

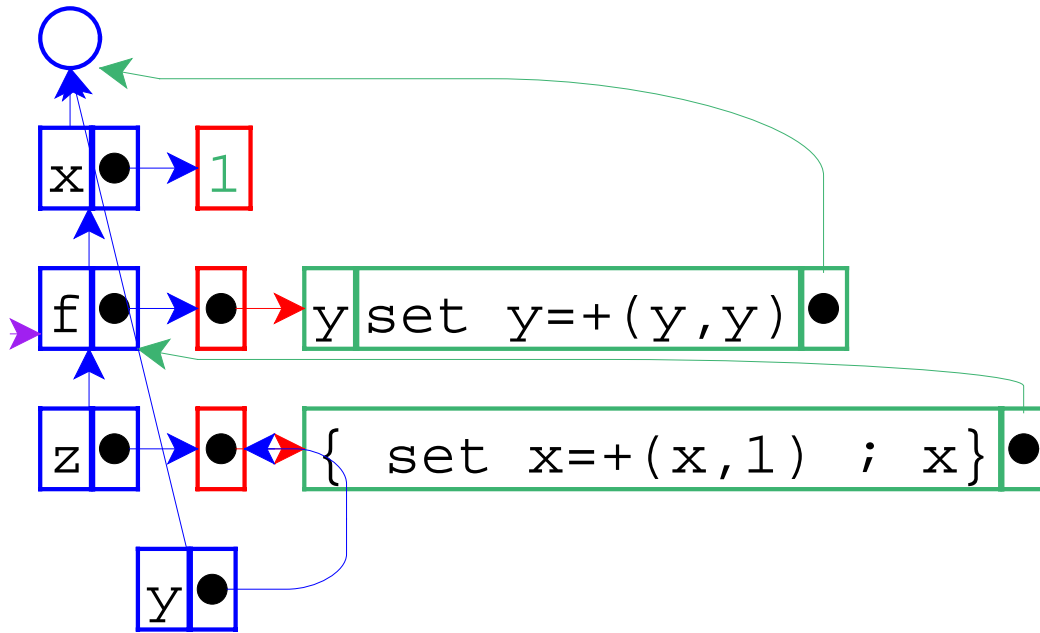
call-by-name/ref



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Result for first `y` is 1

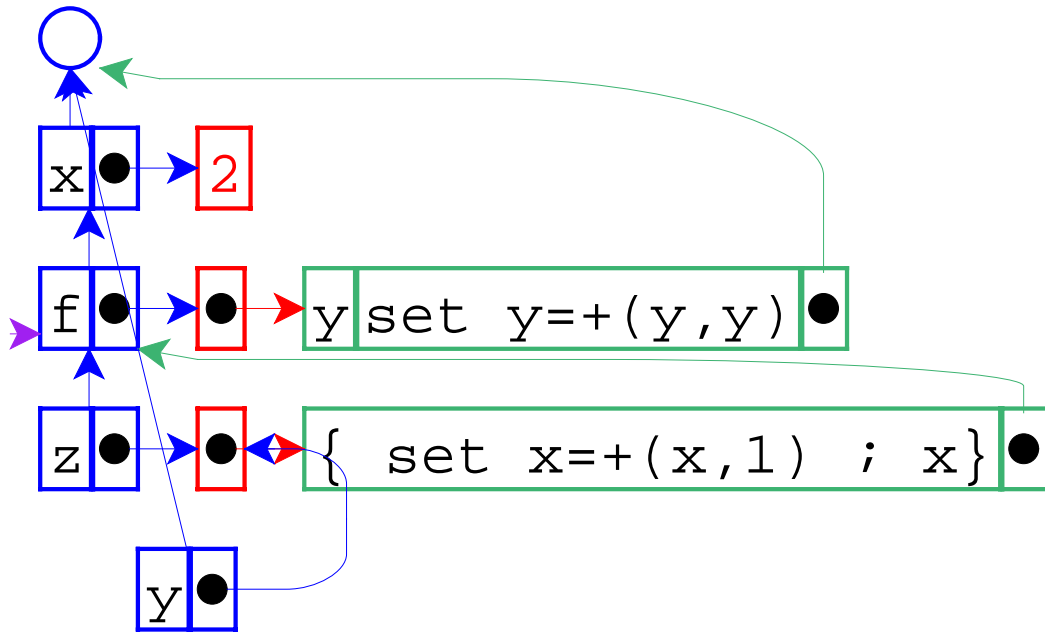
call-by-name/ref



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Second use of `y` triggers evaluation of the thunk

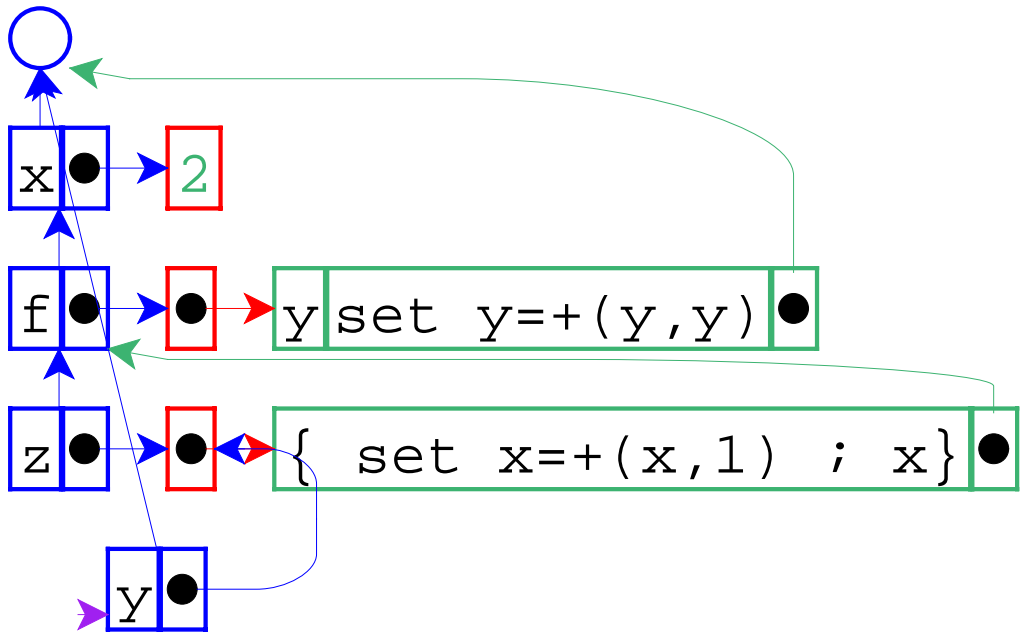
call-by-name/ref



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Think changes value of `x` to 2

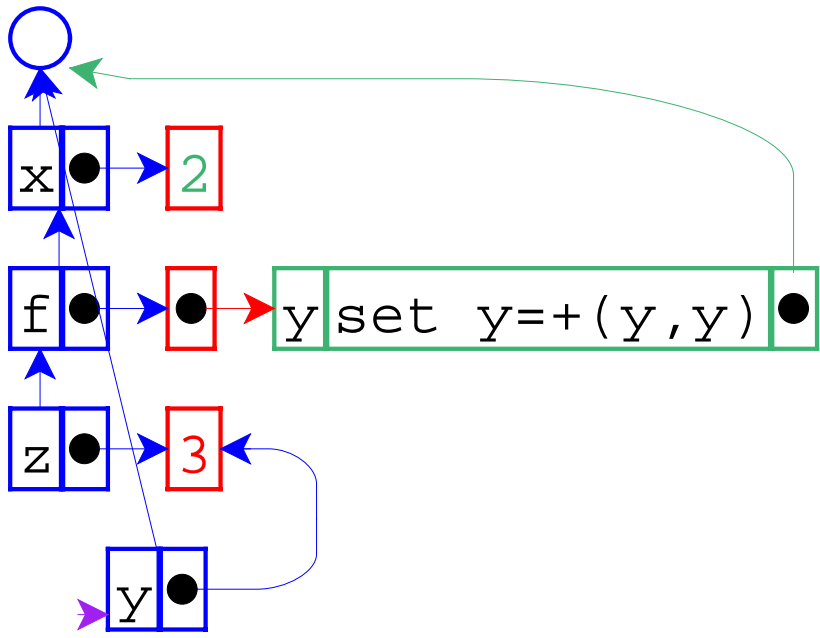
call-by-name/ref



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Result for second y is 2

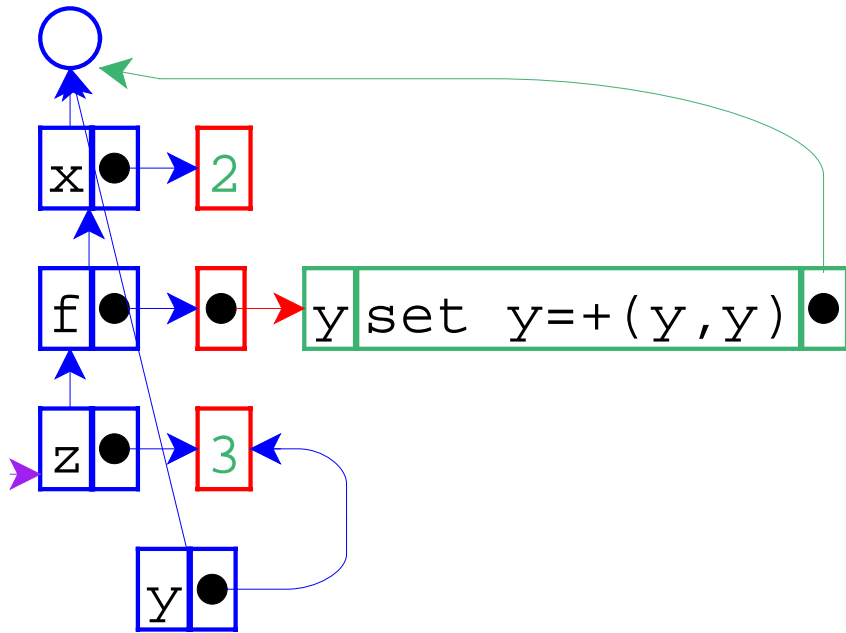
call-by-name/ref



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Set value of y to 3 (= 1+2)

call-by-name/ref



```
let x = 0
  f = proc(y) set y=+(y,y)
in let z = { set x=+(x,1) ; x }
  in { (f z) ; z }
```

- Final result is the value of `z`: 3