Homework 5: Joint Probability, Independence

Instructions: Your answers are due at the beginning of class on the due date. You can either turn in a paper copy, or a pdf version through canvas. We highly recommend using latex (http://www.cs.utah.edu/~jeffp/teaching/latex/) for producing the assignment answers. If the answers are too hard to read (e.g. pdf's from your phone's camera must be very clear and careful) you will lose points (entire questions may be given 0).

Please make sure your name appears at the top of the page.

You may discuss the concepts with your classmates, but write up the answers entirely on your own. Be sure to show all the work involved in deriving your answers! If you just give a final answer without explanation, you may not receive credit for that question.

1. The following data comes from the 2011 Behavioral Risk Factor Surveillance System (BRFSS) survey, which is run by the Centers for Disease Control (CDC). This is a joint probability table for the proportions of survey respondents who smoke and who have had heart attacks.

	Smoker	Non-Smoker
Heart Attack	0.03	0.03
No Heart Attack	0.44	0.50

Answer the following questions:

- (a) What is the proportion of people who don't smoke?
- (b) What is the proportion of people who don't have heart attacks?
- (c) If a person is a smoker, are they more likely to have had a heart attack than someone who is not a smoker? **Hint: Be careful what this question is asking!**
- (d) If you know someone did not have a heart attack, what is the probability that they are not a smoker?
- (e) Is smoking independent of having a heart attack?
- 2. Let k be some constant number, and consider continuous random variables X and Y with joint pdf

$$f(x,y) = \begin{cases} k \sin(2x+y) & \text{for } 0 \le x \le \frac{\pi}{4}, \ 0 \le y \le \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find k.
- (b) What is the joint probability $P\left(\left\{X \leq \frac{\pi}{8}\right\} \cap \left\{Y \geq \frac{\pi}{4}\right\}\right)$?
- (c) What is the marginal pdf $f_X(x)$?
- (d) What is the marginal pdf $f_Y(y)$?
- (e) What is the conditional probability $P\left(X \leq \frac{\pi}{8} \mid Y = \frac{\pi}{4}\right)$?
- (f) Are X and Y independent? Explain why or why not.