Homework 1: Getting Started with Probability

Instructions: Write your answers directly on this pdf (via an editor, iPad, or pen/pencil). The answers should be in the specified place. Students will be responsible for loading their assignments to GradeScope, and identifying what page contains each answer.

The assignment should be uploaded by 11:50pm on the date it is due. There is some slack built into this deadline on GradeScope. Assignments will be marked late if GradeScope marks them late.

If the answers are too hard to read you will lose points (entire questions may be given 0).

Please make sure your name appears at the top of the page.

You may discuss the concepts with your classmates, but write up the answers entirely on your own. Be sure to show all the work involved in deriving your answers! If you just give a final answer without explanation, you may not receive credit for that question.
1. An usher at a wedding asks guests at random whether they are a friend of the bride or the groom. Define the events $B = \text{“friend of the bride”}$ and $G = \text{“friend of the groom.”}$ Write an expression for each of the following events using set operations involving events $B$ and $G$. Here you can just give the answer, and do not need to show any work.

(a) The guest is not a friend of the bride.

(b) The guest is a friend of the bride and not a friend of the groom.

(c) The guest is either a friend of the bride or a friend of the groom (or both).

(d) The guest is not a friend of the bride and not a friend of the groom.

(e) The guest is not a friend of the bride or not a friend of the groom (could be a friend of the bride or the groom, but not both).
2. Let $\Pr(A) = 0.3$, $\Pr(B) = 0.5$ and $\Pr(A \cap B) = 0.25$. Calculate the following probabilities. Be sure to show your intermediate steps and list any probability rules that you use.

(a) $\Pr(A \cup B)$

(b) $\Pr(A^c)$

(c) $\Pr(A^c \cup B^c)$

(d) $\Pr(A \cap B^c)$

(e) $\Pr(A^c \cap B^c)$
3. We have an urn containing three balls labeled 1, 2 and 3. Suppose we draw two balls at random.

(a) What is the sample space for this experiment?

(b) Write down the set of outcomes corresponding to the following events:

A: One of the balls has the label “2”.

B: The sum of the two numbers is 3.

C: The sum of the two numbers is at least 4.

D: The sum of the two numbers is even.
(c) Calculate the probability of each of the following events: $A^c, A \cup (C \cap D), A \cup D^c$. Please provide the following information: (1) The set of outcomes corresponding to each event. (2) The probability of the event, preferably expressed as a fraction.

$\Pr(A^c)$:

$\Pr(A \cup (C \cap D))$:

$\Pr(A \cup D^c)$:
4. You have a deck of 18 cards, with:

- 10 cards that are white on both sides;
- 5 cards that are white on one side, black on the other;
- 3 cards that are black on both sides.

The cards are shuffled and randomly flipped. You draw one card at random from the deck and look only at one side of it. *Hint:* Be careful! Each side of the card counts as a separate outcome. Think about it in terms of the 36 sides that are possible (not the 18 possible cards).

(a) Draw a tree diagram with the probabilities of all possible card types. Your tree should have two levels, one for the color of the top of the card, and one for the color of the bottom of the card. It should also include all of the joint probabilities (top color AND bottom color) at the ends.

(b) What is the probability that the top of your card is white?

c) If the top of your card is black, what is the probability that the bottom is white?

d) What is the probability that the bottom of your card is black? *Show how you got this.*