Prob / Stats L04

Total Probability

Jan 26, 2023

Review: Independent events A, B

$$P_r(A) = P_r(A \mid B)$$

$$P_r(B) = P_r(B \mid A)$$

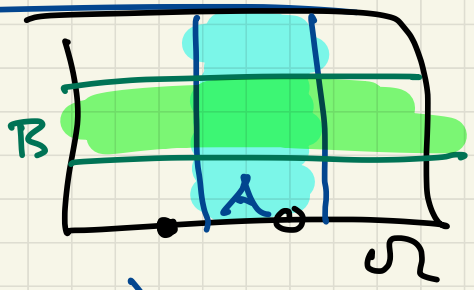
$$P_r(A \cap B) = P_r(A) \cdot P_r(B)$$

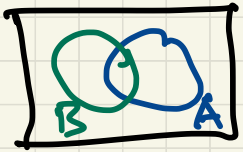
Also

↳ $$P_r(A) = P_r(A \mid B^c)$$

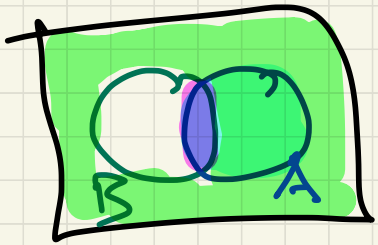
$$P_r(B^c) = P_r(B^c \mid A)$$

$$P_r(A \cap B^c) = P_r(A) \cdot P_r(B^c)$$





$$P_c(A \cap B) = P_c(A) \cdot P_c(B)$$



$$\Leftrightarrow P_c(A - B^c) = P_c(A) \cdot P_c(B)$$

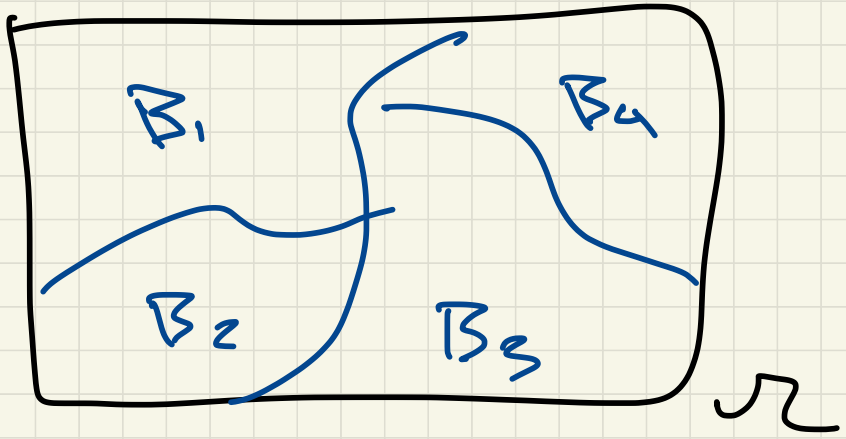
$$\Leftrightarrow P_c(A) - P_c(A \cap B^c) = P_c(A) \cdot P_c(B)$$

$$\Leftrightarrow P_c(A) - P_c(A \cap B^c) = P_c(A)\left(1 - P_c(B^c)\right)$$

$$\Rightarrow P_c(A \cap B^c) = P_c(A)\, P_c(B^c)$$

$$P_c(A \cap B^c) = P_c(A) \cdot P_c(B^c)$$

Total Probability



Partition of set Ω in

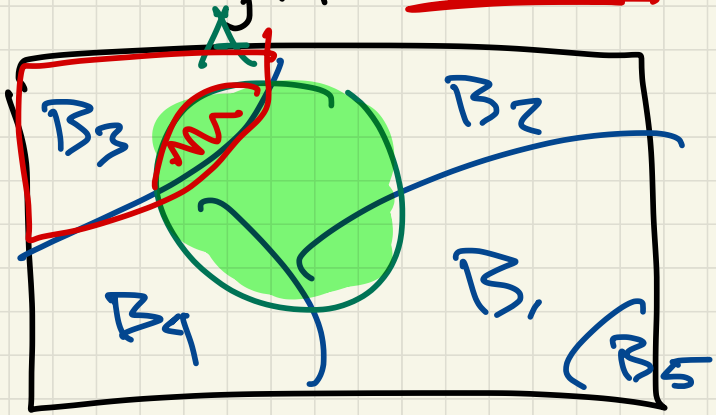
$B_1, B_2, \ldots B_k$

① $B_i \cap B_j = \phi$ disjoint ② $B_1 \cup B_2 \cup \ldots \cup B_k = \bigcup_{j=1}^{k} B_j = \Omega$

Partition $B_1, B_2, \dots B_R$ of Ω

$$P_r(A) = P_r(A|B_1)P_r(B_1) + P_r(A|B_2)\cdot P_r(B_2) + \dots + P_r(A|B_R)P_r(B_R)$$

$$= \sum_{j=1}^{R} P_r(A|B_j)\cdot P_r(B_j)$$



if $A = \Omega$

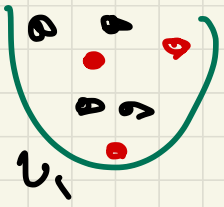
$P_r(\Omega) = 1$

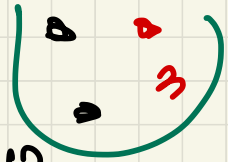
$$\sum_{j=1}^{R} P_r(\Omega|B_j)\cdot P_r(B_j) = \sum_{j=1}^{R} 1 \cdot P_r(B_j)$$

$$P_r(A) = P_r(A \mid B) \cdot P_r(B) + P_r(A \mid B^c) P_r(B^c)$$

$$A = (A \cap B) \cup (A \cap B^c) \quad A \cap (B \cap B^c) = \phi$$

two urns : ① select urn ② select ball from urn





A = red

B = urn 1

1/2

Urn 1

3/7

red

$\frac{1}{2} \cdot \frac{3}{7} = \frac{3}{14}$

4/7

black

$\frac{1}{2} \cdot \frac{4}{7} = \frac{2}{7}$

1/2

Urn 2

1/2

red

$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

1/2

black

$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$Pr(\text{red ball}) = Pr(r \mid U_1) \cdot Pr(U_1) + Pr(r \mid U_2) \cdot Pr(U_2)$

$= 3/14 + 1/4 = 13/28$

Two Events A, B

A	B
1	1
1	0
0	1
0	0

Probability $P : 2^{\Omega} \to [0,1]$

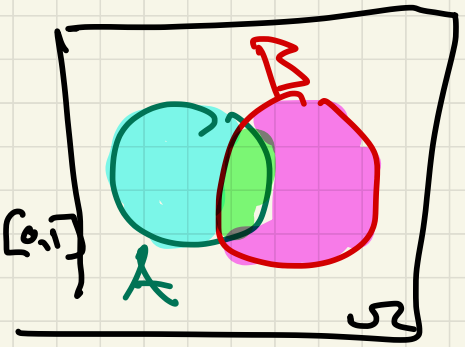
$\to 0.1$

$\to 0.2$

$\to 0.3$

$\to 0.4$

1.0



A

B	True	False	
True	0.1	0.3	0.4
False	0.2	0.4	0.6
	0.3	0.7	