Notes: Total Probability

CS 3130/ECE 3530: Probability and Statistics for Engineers

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A set of events $B_1, B_2, \ldots, B_n$ is a **partition** of Ω if they are pairwise disjoint, that is, $B_i \cap B_j = \emptyset$ for any i, j, and if their union is equal to all of Ω, that is, $B_1 \cup B_2 \cup \ldots \cup B_n = \Omega$.

Given a partition $B_1, B_2, \ldots, B_n$ of Ω, the **law of total probability** states

$$P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + \ldots + P(A|B_n)P(B_n)$$

A common application of this rule is for <u>any</u> event B, where we will have B, B^c forming a partition of Ω. Here the total probability is just two terms:

$$P(A) = P(A|B)P(B) + P(A|B^c)P(B^c)$$

<u>In-Class Problem:</u> You have two urns, one with 4 black balls and 3 white balls, the other with 2 black balls and 2 white balls. You pick one urn at random and then select a ball from the urn. What is the probability the ball is white?

<u>In-Class Problem:</u> You have a system with a main power supply and auxiliary power supply. The main power supply has a 10% chance of failure. If the main power supply is running, the auxiliary power supply also has a 10% chance of failure. But if the main supply fails, the auxiliary supply is more likely to be overloaded and has a 15% chance to fail. What is the probability that the auxiliary power will fail?