CS7960 L22 : GPU | Sorting

GPU

Parallel processor
- Many cores
- Small memory

memory transfer overhead

Sorting:
Input: Large array A = <a1, a2, ..., an>
Output B = <b1,b2,...,bn>
 - mu(a_i) = b_j exists
 - b_j <= b_{j+1}

Data driven sorting?
 - insertion sort?
   O(n^2)
  (choose one and place in correct spot)
 - quick sort?
   O(n log n)
  (need splitter: median hard, otherwise varies size...)
 - heap sort?
   O(n log n)
   (need to maintain heap data structure, hard on GPU)
 - radix sort?
   O(nk) (for k digit w/ constant bits)
   lengths of each digit category uncontrollable length.

<hard to make highly parallel>

Data Independent sorting
 - bubble sort?
   O(n^2)
   (compare all neighbors)
   very parallelizable, but takes n rounds to move point from 1 to

n
- merge sort?
  O(n log n)
  (divide + conquer + join)
  join step very sequential :(
- bitonic sort
  (divide + conquer + join)
  join step parallel !!!

<will also hybridize merge+bubble...>

Bitonic Sort:

Bitonic sequence:

```
 - increasing,                          1 2 4 6 8 11
 - decreasing,                          9 7 4 3 2 1
 - increasing then decreasing, or      1 4 6 9 3 2
 - decreasing then increasing.          9 5 2 3 4 6
 (at most one local maxima/minima)
```

BitonicSplit(A):
Input: 1 bitonic sequence A size n
Ouput: 1 increasing (sorted) sequence B size n

```
for h = log n to 1
 for i = 1 to n/2^h PARDO
  for j = 0 to 2^{h-1} PARDO
   min(A[i + (2j)*(n/2^h)], A[i + (2j+1)(n/2^h)]) -> B[i + (2j)*(n/2^h)]
   max(A[i + (2j)*(n/2^h)], A[i + (2j+1)(n/2^h)]) -> B[i + (2j+1)(n/2^h)]
```

Example:

```
24 20 15  9  4  2  5  8|10 11 12 13 22 30 32 45
10 11 12  9| 4  2  5  8|24 20 15 13|22 30 32 45
 4  2| 5  8|10 11|12  9|22 20|15 13|24 30|32 45
 4| 2| 5  8|10  9|12 11|15 13|22 20|24 30|32 45
 2  4  5  8  9 10 11 12 13 15 20 22 24 30 32 45
```

How to get a bitonic sequence?

```
for h = 1 to log n
 for i = 1 to n/2^h PARDO
  for j = 0 to 2^{h-1} PARDO
   BitonicSplit(A[i + (2j)(n/2^h), i + (2j+2)(n/2^h) - 1])  //
(reverse second half)
```

 - sets of size 2 are bitonic
 - let S be an ascending sorted set
   let T be a descending sorted set
   S cat T is bitonic
 - run bitonic sort of sets of doubled size for log n rounds

BitonicSplit on all pairs -> sort all pairs
BitonicSplit on all quads (reverse second pair) -> sort all quads
...
BitonicSplit on list (reverse second half) -> sorted list

O(log n) rounds of Bitonic split
 Each Bitonic split takes O(log n) rounds

O(log^2 n) parallel time
O(n log^2 n) work

Fine-grain parallelism:
 - core of each operation is a compare.
 - data independent

For several years, this was fastest GPU sort!
What are the weak points of this?
How can it be improved?

Hybrid (bucket/quick + merge sort)

Sintorn + Assarsson 08
(beats bitonic by factor 2-3)
takes advantage of advanced architecture of GPU (GeForce 8800)

1. Create L sub-lists using L-1 {l_1,l_2,...l_{L-1}} pivotes
   so p in Li has l_i < p <= l_{i+1}
2. Move each L_i to separate processor group
3. Merge Sort on each list L_i

details:
(1) three proposed methods:
 (a) bucket sort (two-rounds)
     i : choose L-1 pivots by linear interpolation [min,max]
         (random sample may work better, distribution independent)
     ii : build histogram w/ AtomicInc on buckets
     iii: re-linear interpolate based on histogram
         (again I think random sample may work better, more general)
 (b) Use NVidia histogram functionality to help w/ splits.
 (c) Run log(L) rounds of quick sort by choosing random pivots

 (d) other option: run multi-selection sort we discussed in class
              or just log(L) median operations in O(N) time each

Note: assigning a point p to a pivot can be done in parallel, but takes O(log L) (binary search on {l_i}_i). Perhaps can be done quicker with clever bit-shifting....

(2) Use local hierarchy of GPU to move to sub-hierarchies on GPU each L of roughly the same size.
Importance of same size, otherwise, when last is running, others will be idle.

(3)
1. break to sets of size 4
2. run special "kernel" to sort sets of size 4
3. merge pairs of sets
     (for most of run, many more sets than processors, so highly parallel)
4. eventually p processors in group, and < p lists left to merge
     (lose some parallelism, but oh,well, did pretty well).

Work = O(n log n)

```
PTime :
 (1) = O(log L)
      (a) 2 rounds of O(log L) time to assign
      (c) log L rounds of finding median (and counting)
           * O(log n log log n) to find median
          but heuristic (random split) only takes O(1)/round
 (2) = O(log L)  (each list of size roughly N/L)  (but could be
N !)
 (3) = O(n/L) since last round one 1 processor needs to run a
merge on two lists.
     = O(n/L + log L) optimal for L = n --> (log n)
       but that requires (1) to complete sort!  ...L restricted by
num processors
```

Odd-Even Transition Merge Sort:

Odd-Even Transition Sort:

```
for h = 1 to n/2
 for i=1 to n/2 PARDO
  min(A[2i-1],A[2i]) -> A[2i-1]
  max(A[2i-1],A[2i]) -> A[2i]
 for i=1 to n/2-1 PARDO
  min(A[2i],A[2i+1]) -> A[2i]
  max(A[2i],A[2i+1]) -> A[2i+1]
```

O(n) Ptime, O(n^2) Work

Way to make this
 - O(log^2 n) Ptime
 - O(n log^2 n) Work
 - fine-grained
 - data independent

1. Grow sorted sub-pieces
2. Join takes O(log m) for sorted sets of size m

"sorting network"