

## CS7960 L13 : Parallel I (Prefix) Sum

PRAM

1 disk  
P processors  
n input items

Each time step a processor can:  
read, write, operate (+,-,\*,<<,...)

shared memory: CRCW (although CREW more realistic)

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Sum (n):  
INPUT A = [a<sub>1</sub>, a<sub>2</sub>, ..., a<sub>n</sub>]

Sequential?  $O(n)$

PRAM:  $O(n/p + \log n)$

```
#####  
for i=1 to n PARDO  
  B(0,i) := A(i)  
for h = 1 to log n DO  
  for i=1 to n/2^h PARDO  
    B(h,i) := B(h-1,2i-1) + B(h-1,2i)  
return B(log n, 1)  
#####
```

(log n) rounds:  
A=B<sub>0</sub> = 7 4 2 5 1 4 9 2  
B<sub>1</sub> = 11 7 5 11  
B<sub>2</sub> = 18 16  
B<sub>3</sub> = 34

$O(n)$  work,  $O(\log n)$  time

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PRAM = BSP

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Prefix Sum

INPUT A = [a<sub>1</sub>, a<sub>2</sub>, ..., a<sub>n</sub>]  
 OUTPUT B = [a<sub>1</sub>, a<sub>1</sub>+a<sub>2</sub>, a<sub>1</sub>+a<sub>2</sub>+a<sub>3</sub>, ...]  
 b<sub>i</sub> = sum\_{j=1}^i a<sub>j</sub>

Sequential? O(n)

```

#####
for i=1 to n PARDO
  B(0,i) := A(i)
for h = 1 to log n DO
  for i=1 to n/2^h PARDO
    B(h,i) := B(h-1,2i-1) + B(h-1,2i)
for h = log n to 0 DO
  for i=1 to n/2^h, even PARDO
    C(h,i) := C(h+1,i/2)
  C(h,1) := B(h,1)
  for i=3 to n/2^h, odd PARDO
    C(h,i) := C(h+1, (i-1)/2) + B(h,i)
Output C (PAROUT)
#####

```

Builds sum, then distributes back down.

log n rounds up, log n rounds down.

(log n) rounds:

```

A=B0 = 7 4 2 5 1 4 9 2
B1   = 11 7 5 11
B2   = 18 16
B3   = 34
C3   = 34
C2   = 18 34
C1   = 11 18 23 34
C0   = 7 11 13 18 19 23 32 34

```

O(n) work, O(log n) time