

FoDA L6

# Convergence

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Central Limit Theorem


and  
Estimation

Sep 8, 2022

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## Wellness

Personal concerns such as stress, anxiety, relationship difficulties, depression, cultural differences, etc., can interfere with a student's ability to succeed and thrive at the University of Utah. For helpful resources contact the Center for Student Wellness at [www.wellness.utah.edu](http://www.wellness.utah.edu) or 801-581-7776.

If you are a student veteran, the U of Utah has a Veterans Support Center located in Room 161 in the Olpin Union Building. Hours: M-F 8-5pm. Please visit their website for more information about what support they offer, a list of ongoing events and links to outside resources: <http://veteranscenter.utah.edu/>.

Contact the instructor if there is any additional support that would aid in this course.

# Sampling of Data

n data points  
observations

$$P = \{p_1, p_2, \dots, p_n\}$$

fixed

drawn iid

from unknown  
distribution  $f$

iid : Independently and Identically Distributed.

likelihood  $f(D|M) = \prod_{i=1}^n g(x_i)$

Random Variable

$$X \sim f$$

estimate

$$\bar{P} = \frac{1}{n} \sum_{i=1}^n P_i$$

sample mean

observations

$$P = \{P_1, \dots, P_n\}$$

random variables

$$\{X_1, X_2, \dots, X_n\}$$

random variable

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$$

How good

will

$$\bar{P}$$

represent

mean

$$\left( \text{or } E[X] \right)$$

?

$$\bar{P} = \frac{1}{n} \sum_{i=1}^n \{P_i\}$$

data

←

RV.  $X \sim f$

← realize  $\{X_i\}$

iid  $f$

# Central Limit Theorem

$n$  iid R.V.s  $X_1, \dots, X_n$  s.t.  $X_i \sim f$

( $f$  : have mean  $\mu = E[X_i]$ , bounded  $\sigma^2$ )

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$$

1.  $\bar{X}$  converges to normal distribution as  $n \rightarrow \infty$

2.  $\mu_{\bar{X}}$  (of  $\bar{X}$ ) =  $E[X_i] = \mu$

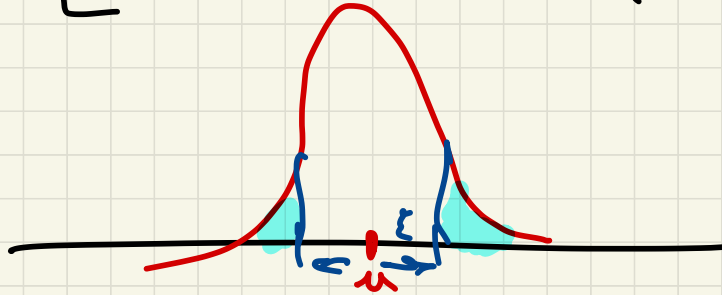
3. Variance  $\sigma_{\bar{X}}^2 = \sigma^2 / n$

# Probably Approximately Correct PAC

R.V.  $\bar{x}$

$$P_r \left[ |\bar{x} - E[\bar{x}]| > \epsilon \right] \leq \delta$$

*error tolerance*



prob. of failure