FoDA L6

Convergence

Central Limit Theorem

and

Estimation

Sep 8, 2022

Wellness

Personal concerns such as stress, anxiety, relationship difficulties, depression, cultural differences, etc., can interfere with a students ability to succeed cross and thrive at the University of Utah. For helpful resources contact the Center for Student Wellness at www.wellness.utah.edu or 801-581-7776.

If you are a student veteran, the U of Utah has a Veterans Support Center located in Room 161 in the Olpin Union Building. Hours: M-F 8-5pm. Please visit their website for more information about what support they offer, a list of ongoing events and links to outside resources: http://veteranscenter.utah.edu/.

Contact the instructor if there is any additional support that would aid in this course.

Sampling of Data

n data points $\quad P = \{p_1, p_2, \ldots, p_n\}$

observations $\quad$ fixed

drawn iid from unknown distribution f

iid : Independtly and Identially Distributed.

likelihood $\quad f(D \mid M) = \prod_{i=1}^{n} g(x_i)$

Random Variable $X \sim f$

observations

$P = \{p_1 \dots p_n\}$

estimate $\bar{P} = \frac{1}{n} \sum_{i=1}^{n} p_i$

sample mean

random variables

$\{X_1, X_2 \dots X_n\}$

random variable $\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$

How good will $\bar{P}$ represent mean (or $E_{X \sim f}[X]$) ?

algo   data   R.V.

$\bar{P} \xleftarrow{\text{algo}} \{p_i\} \xleftarrow{\text{realize}} \{X_i\} \underset{\text{iid}}{\sim} f$

$= \frac{1}{n} \sum$

Central Limit Theorem

n iid R.V. s $X_1, \ldots X_n$ s.t. $X_i \sim f$

(f : have mean $\mu = E[X_i]$, bounded σ^2)

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$$

1. $\bar{X}$ converges to normal distribution as $n \to \infty$

2. $\mu_{\bar{X}}$ (of $\bar{X}$) $= E[X_i] = \mu$

3. variance $\sigma_{\bar{X}}^2 = \sigma^2 / n$

Probably Approximately Correct

PAC

R.V. $\bar{X}$

error tolerance

$$\Pr\left[\,|\bar{X} - E[\bar{X}]| > \varepsilon\,\right] \le \delta$$

prob. of failure