FoDA L2

Probability Review #1

Events, Random Variables, Independence

8.25.22

Students Support and Inclusion

Students with Disabilities or with English as a non-First Language

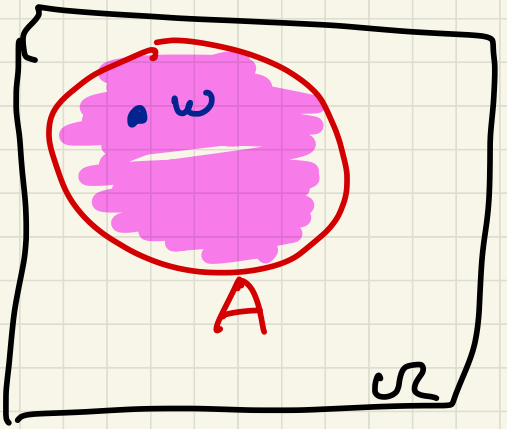
The University of Utah seeks to provide equal access to its programs, services, and activities for people with disabilities. If you need accommodations in this class, reasonable prior notice needs to be given to the Center for Disability Services, 162 Olpin Union Building, 581-5020. CDS will work with you and the instructor to make arrangements for accommodations. All written information in this course can be made available in an alternative format with prior notification to the Center for Disability Services.

Extra support if also available for those for whom English is not their first language. Several resources on campus will support you with your language and writing development. These resources include: the Writing Center (http://writingcenter.utah.edu/); the Writing Program (http://writing-program.utah.edu/); the English Language Institute (http://continue.utah.edu/eli/). Contact the instructor if there is any additional support that would aid in this course.

Sample Spaces Ω

$\omega \in \Omega$ "omega in Omega" "omega not in Ω"

sample outcome



$A \subseteq \Omega$ "A is subset Omega"

event subset of

6-sided die $\Omega = \{1, 2, 3, 4, 5, 6\}$

ex $\omega = 3$

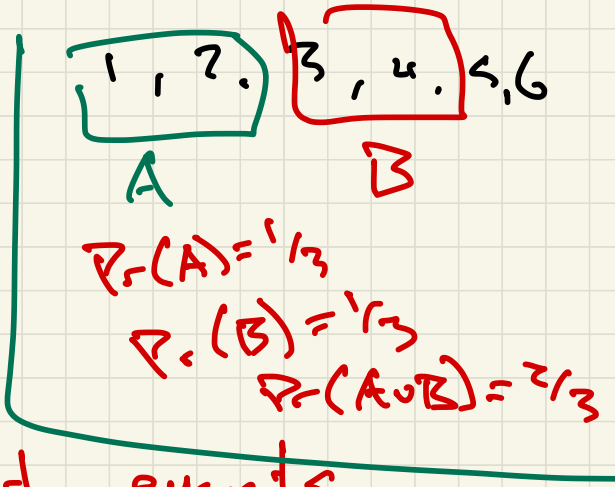
$A = \{1, 3, 5\}$ odd

$Pr(A)$ - probability of A

$= 1/2$

Probability $Pr(A)$

- $0 \le Pr(A) \le 1$
- $Pr(\Omega) = 1$
- total prob.



disjoint events

$A_1, A_2, \ldots$

$A_i \wedge A_j = \phi$

$$Pr\left(\bigcup_{i=1} A_i\right) = \sum_{i=1} Pr(A_i)$$

Biased Coin

$A_1 = T$ $\quad Pr(T) = 0.4$

$A_2 = H$ $\quad Pr(H) = 0.6$

$\Omega = \{H, T\}$

$Pr(H) + Pr(T) = 1$

Continuous Sample Spaces

- grade percentage
- rainfall
- time

ex Start class at 10:45

$\Omega : [10:45:00, 10:46)$

event $A : [10:45:00, 10:45:30]$

$Pr(A) = 0.8$

Random Variable X, Y, Z.

random process → outcome → assigned value

role dice → face of die → value $\{1, 2, 3, 4, 5, 6\}$

$\mathbb{R}$ real values

$\mathbb{R}^d$ d-dimension real vectors

R.V. $D_1, D_2, D = D_1 + D_2$

Two events A, B

cond. prob. $P(A \mid B)$ ← given = conditioned on B being true.

One die

B = die even $\{2, 4, 6\}$ $Pr(B) = \frac{1}{2}$

A = die value ≤ 3 $\{1, 2, 3\}$

$Pr(A) = 1/2$ $Pr(A \mid B) = 1/3$

$$Pr(A \mid B) = \frac{Pr(A \cap B)}{Pr(B)} \qquad \frac{1/6}{1/2} = 1/3$$

$+ Pr(B \mid A)$

Independence? $Pr(A \mid B) = Pr(A)$

or $Pr(B \mid A) = Pr(B)$ or $Pr(A) \cdot Pr(B) = Pr(A \cap B)$

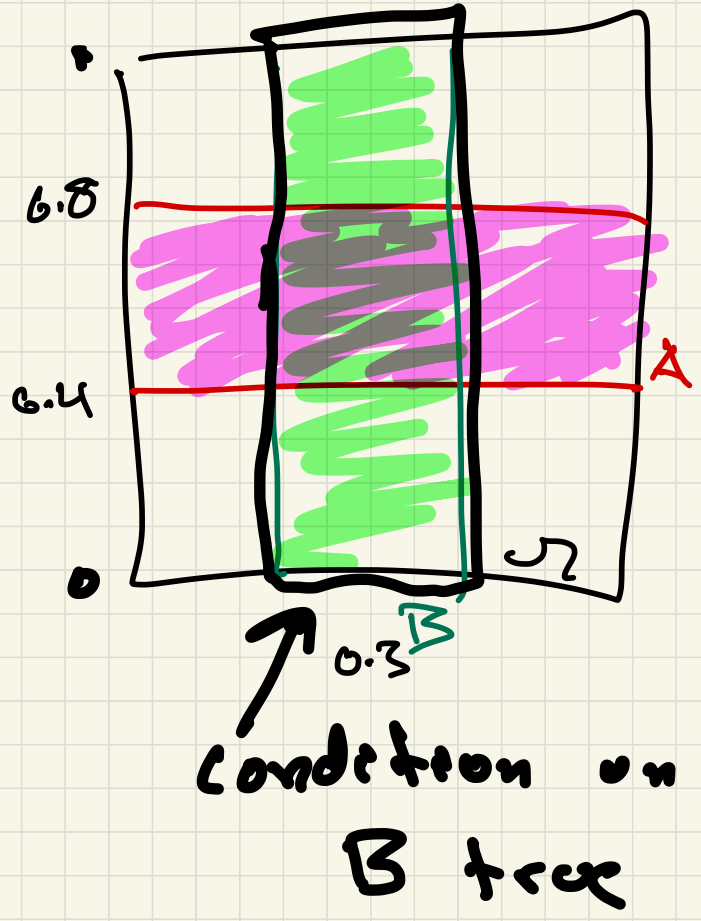
$P_r(A \mid B)$

$P_r(A) = 0.4$

$P_r(B) = 0.3$

$P_r(A) \cdot P_r(B) = 0.12$

$P_r(A \cap B) = 0.12$



2 Random Variable X, Y

Independent

if all events $A \subseteq \Sigma_X$

$B \subseteq \Sigma_Y$

$$Pr(A \cap B) = Pr(A) \cdot Pr(B)$$