CS 3190: Foundations of Data Analysis **Due: Wednesday 10.07 at 1:10pm**

Homework 2: Convergence and Linear Algebra

Instructions: Your answers are due at 1:10, before the beginning of class on the due date. You must turn in a pdf through canvas. I recommend using latex (`http://www.cs.utah.edu/~jeffp/teaching/latex/`, see also `http://overleaf.com`) for producing the assignment answers. If the answers are too hard to read you will lose points, entire questions may be given a 0 (e.g. **sloppy pictures with your phone's camera are not ok, but very careful ones are**)

Please make sure your name appears at the top of the page.

You may discuss the concepts with your classmates, but write up the answers entirely on your own. **Be sure to show all the work involved in deriving your answers! If you just give a final answer without explanation, you may not receive credit for that question.**

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1. **[30 points]** Consider a random variable X with expected values $\textbf{E}[X] = 100$ and variance $\textbf{Var}[X] = 144$. We would like to upper bound the probability $\textbf{Pr}[X < 75]$.

   (a) Which bound can and cannot be used with what we know about X (Markov, Chebyshev, or Chernoff-Hoeffding), and why?

   (b) Using that bound, calculate an upper bound for $\textbf{Pr}[X < 75]$.

   (c) Describe a probability distribution for X where the other two bounds are definitely not applicable.

2. **[30 points]** Consider n iid random variables $X_1, X_2, \ldots, X_n$ with expected value $\textbf{E}[X_i] = 7$ and variance $\textbf{Var}[X_i] = 2$. Assume we also know that each X_i must satisfy $1 \leq X_i \leq 13$. We now want to analyze the random variable of their average $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$.

   Assume first that $n = 2$ (the number of random variables).

   (a) Use the Chebyshev inequality to upper bound $\textbf{Pr}[\bar{X} > 12]$.

   (b) Use the Chernoff-Hoeffding inequality to upper bound $\textbf{Pr}[\bar{X} > 12]$.

   Now assume first that $n = 20$ (the number of random variables).

   (c) Use the Chebyshev inequality to upper bound $\textbf{Pr}[\bar{X} > 12]$.

   (d) Use the Chernoff-Hoeffding inequality to upper bound $\textbf{Pr}[\bar{X} > 12]$.

3. **[15 points]** Consider the following 2 vectors in $\mathbb{R}^4$:

$$
\begin{aligned}
p &= (1, -2, 4, \mathtt{x}) \\
q &= (2, -4, 8, -2)
\end{aligned}
$$

   Report the following:

   (a) Choose the value $\mathtt{x}$ so that p and q are linearly dependent

   (b) Choose the value $\mathtt{x}$ so that p and q are orthogonal

   (c) Calculate $\|q\|_1$

(d) Calculate $\|q\|_2^2$

4. **[25 points]** Consider the following 2 matrices:

$$A = \begin{bmatrix} 2 & -1 & 4 \\ 0 & -1 & 0 \\ 3 & -2 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix}$$

Report the following:

(a) $A^T B$

(b) AB

(c) BA

(d) $B + A$

(e) B^T

(f) Which matrices are invertable? For any that are invertable, report the result.