FoDA

L3

Probability

Review

#2

Probability Density Functions (pdf)

continuous RV $X : \Lambda \to \Omega = \mathbb{R}$

$$f_X : \mathbb{R} \to \mathbb{R}_{\geq 0}$$

$$P_r(X \in A) = \int_{\omega \in A} f_X(\omega)\, d\omega$$

event $A \subset \Omega$

cumulative density function (cdf)

$$F_X(t) = \int_{\omega=-\infty}^{t} f_X(\omega)\, d\omega = P_r(X \in A_t)$$

$A_t = (-\infty, t]$

$\in [0,1]$

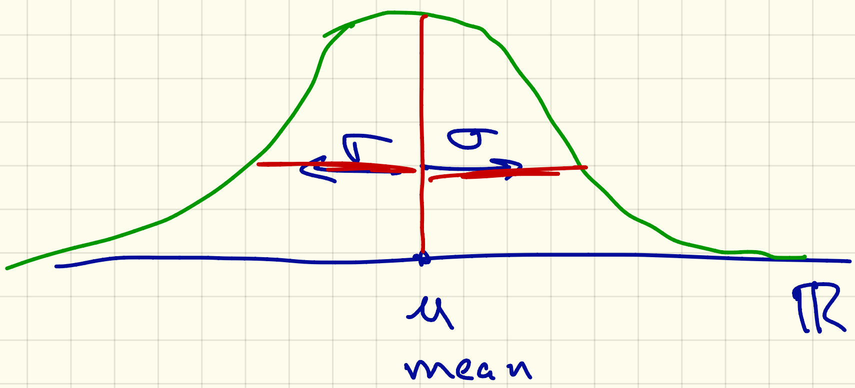
$$f_X(\omega) = \frac{dF_X(\omega)}{d\omega}$$

Normal Random Variable

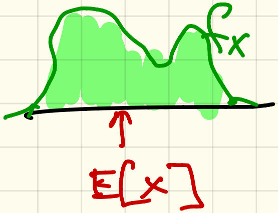
$$X \sim N(\mu, \sigma)$$

$$f_X(\omega) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{(\omega-\mu)^2}{2\sigma^2}\right)$$

$$\exp(x) = e^x$$



Expected Value



R.V. $X : \Lambda \to \Omega = \mathbb{R}$

discrete $E[X] = \sum_{\omega \in \Omega} \left(\omega \cdot \Pr[X = \omega] \right)$

$\sum (\downarrow) = 1$

continuous $E[X] = \int_{\omega \in \Omega} \omega \cdot f_X(\omega)\, d\omega$

$\int (\downarrow)\, d\omega = 1$

fair 6-sided die $\Omega = \{\underset{\omega_1}{1}, \underset{\omega_2}{2}, \dots, \underset{\omega_6}{6}\}$

R.V. X $\Pr[X = i] = \frac{1}{6}$ for $i = 1 \dots 6$

$$E[X] = \sum_{\omega_i \in \Omega} \omega_i \cdot \Pr[X = \omega_i] = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + \dots 6 \cdot \frac{1}{6}$$

$$= \frac{21}{6} = 3.5$$

Linearity of Expectation

R.V.s X, Y scalar α

$$E[\underbrace{\alpha X + Y}_{Z}] = \alpha E[X] + E[Y]$$

$E[H] = 1.755\,m$

Average Height

barefoot height f_H $H \sim N(1.755\,m, \sigma = 0.1\,m)$

Shoe height

$S =$ 1cm	2cm	3cm	4cm
0.1	0.1	0.5	0.3

$(0.1)1 + (0.1)(2) + (0.5)3 + (0.3)\cdot 4$

$$E[X_{cm}] = E[H \cdot 100 + S] = 100 \cdot E[H] + E[S]$$

$$= 175.5\,cm + 3\,cm$$

$$= 178.5\,cm$$

Variance

R.V. X

$$\mathrm{Var}[X] = \underset{\Sigma}{E}\left[\left(X - \boxed{E[X]}\right)^2\right]$$

fixed quantity

$$= E[X^2] - \left(E[X]\right)^2$$

$$E[(X - E[X])^2] = E\left[X^2 - 2X \cdot E[X] + E[X]^2\right]$$

$$= E[X^2] - 2E[X]E[X] + E[X]^2$$

$$= E[X^2] - E[X]^2$$

R.V. X, scalar α $\quad \mathrm{Var}[\alpha X] = \alpha^2 \mathrm{Var}[X]$

Standard Deviation

$$\sigma_X = \sqrt{Var[X]}$$

Shoes

$S=1$	$S=2$	$S=3$	$S=4$
0.1	0.1	0.5	0.3

$$E[S] = 3$$

$$Var[S] = E[(S-E[S])^2]$$

$$= \sum_{s=1..4} Pr[S=i] \cdot (i-3)^2 = (0.1)\overset{4}{(1-3)^2} + (0.1)\overset{1}{(2-3)^2}$$

$$+ (0.5)\underset{0}{(3-3)^2} + (0.3)\underset{1}{(4-3)^2}$$

$$= 0.1(4) + 0.1(1) + 0 + (0.3)($$

$$= 0.8$$

$$\sigma_X = \sqrt{0.8} \approx 0.894$$

Covariance

X, Y both R.V.

$$\mathrm{Cov}[X, Y] = E\left[(X - E[X])(Y - E[Y])\right]$$

Joint R.V. X, Y

joint pdf $f_{X,Y} : \Omega_X \times \Omega_Y \to [0, \infty)$

discrete $f_{X,Y}(x, y) = \Pr(X = x,\ Y = y]$

marginal pdf

$$f_X(x) = \sum_{y \in \Omega_Y} f_{X,Y}(x, y) = \sum_{y \in \Omega_Y} \Pr[X = x,\ Y = y]$$

continuous

$$f_X(x) = \int_{y \in \Omega_Y} f_{X,Y}(x, y)\, dy$$

marginal cdf $F_{X,Y}(x, y) = \Pr[X \leq x,\ Y \leq y]$

Random X, Y

independent iff $f_{X,Y}(x,y) = f_X(x) \cdot f_Y(y)$

<u>conditional distribution</u>

$$f_{X|Y}(x \mid y) = \Pr[X \mid Y = y]$$

$$= \frac{f_{X,Y}(x,y)}{f_Y(y)}$$