L22: (Differential) Privacy

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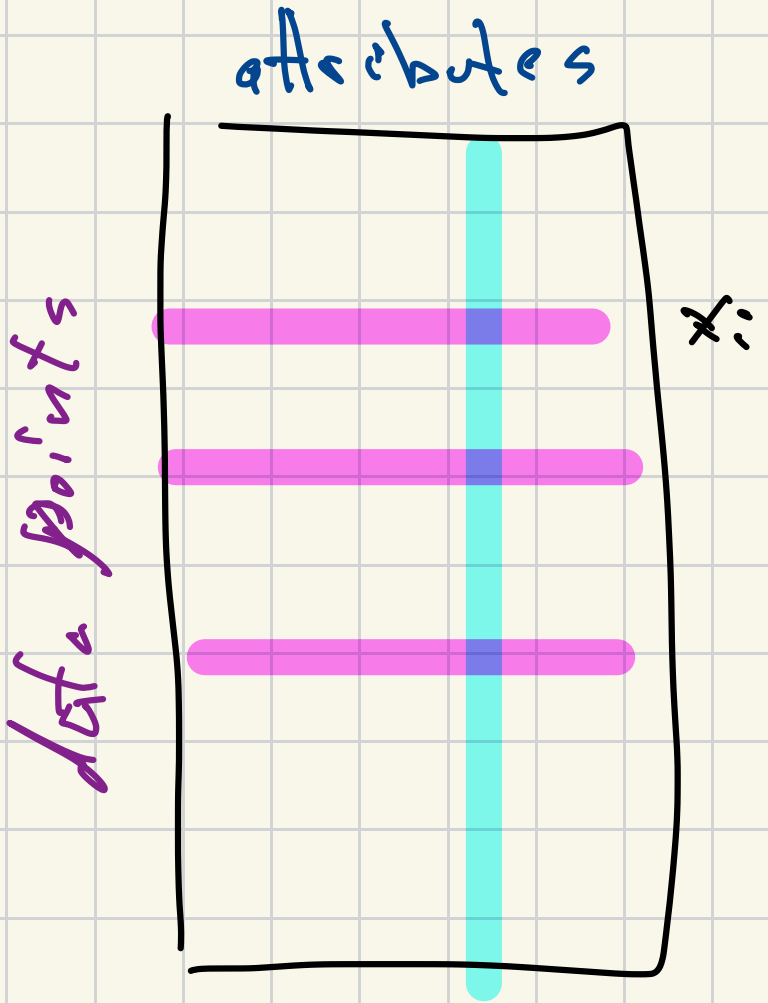
Data X



What if this was you?

Ethics ≈ Empathy

Privacy and Data
- who gets access?
- to what?

Dawn of Data Science

→ Early 2000s

big companies

collect lots data about users

(thru web)

↳ Release sample data online

→ let academia figure out.

Late 2000s this stopped.

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- **Dr.** Sweeney now teaches at Harvard.

How to release data anonymously?

- to keep anonymity individuals
- be able to make inference

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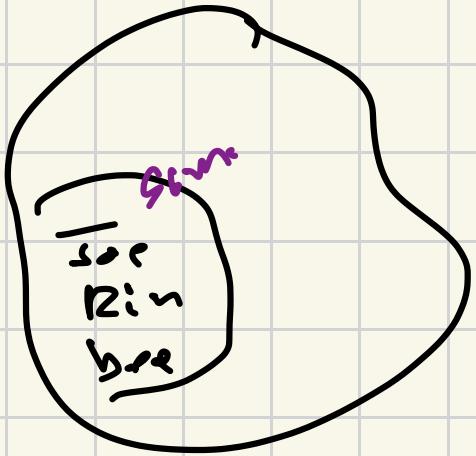
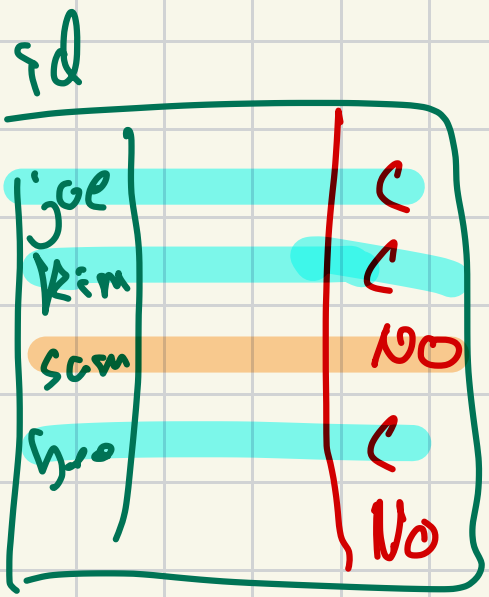
k-anonymity: Remove information from records, until each combination of attributes matches at least k individuals.

Dividing Data

public attributes

- Zip code
- age
- gender





private attributes

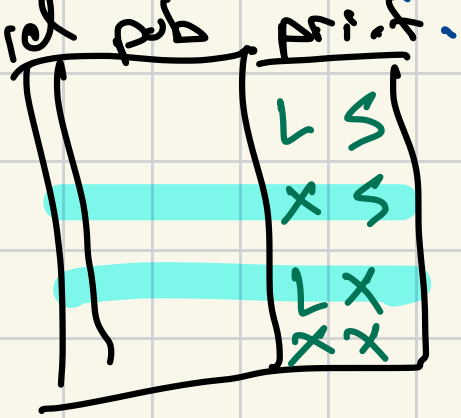
- has cancer
- has diabetes
- search history

l-diversity: k-anonymity ⊕

each group has diversity in private values

What if multiple private attributes?

- lung cancer
- skin cancer



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t-closeness ⊕ l-diversity ⊕

the private distribution values close full data set.

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  And another set $D_2 = \{\langle \text{user-id, movie, date of grade} \rangle\}$.
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- Netflix Prize had proposed sequel, dropped in 2010 for more privacy concerns.

Differential Privacy (DP)

Guarantee on release data X_1

so if another dataset X_2

$$\text{diff}(X_1, X_2) = |X_1 \Delta X_2| = 1$$

$$\frac{\Pr[q(X_1) = v]}{\Pr[q(X_2) = v]} \leq \exp(\varepsilon) \approx 1 + \varepsilon$$

$$= e^{\varepsilon}$$

$$\varepsilon = 0.10$$

$$\Rightarrow \varepsilon\text{-DP}$$

Want to hold for class of queries $\mathcal{Q}$

$q \in \mathcal{Q}$

$$\frac{\Pr[g(X_1)=r]}{\Pr[g(X_2)=r]} = 2$$

$1+\varepsilon$

$\Rightarrow \varepsilon = 1$

$\Rightarrow$ twice as likely to predict X_1 than X_2

$$\frac{\Pr(g(X_1)=r]}{\Pr(g(X_2)=r]} = 99$$

Use $\varepsilon \in (0, \infty)$

usually $\varepsilon \approx 1$ OR

Data $X \in \mathbb{R}$ (ex. height of Sly Stallone)

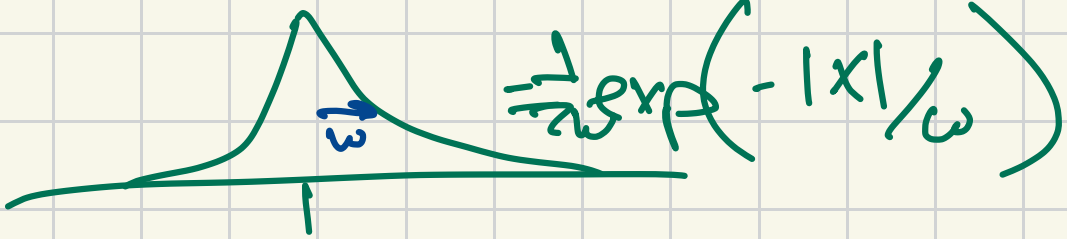
X'

$x_1 = 68$ inches

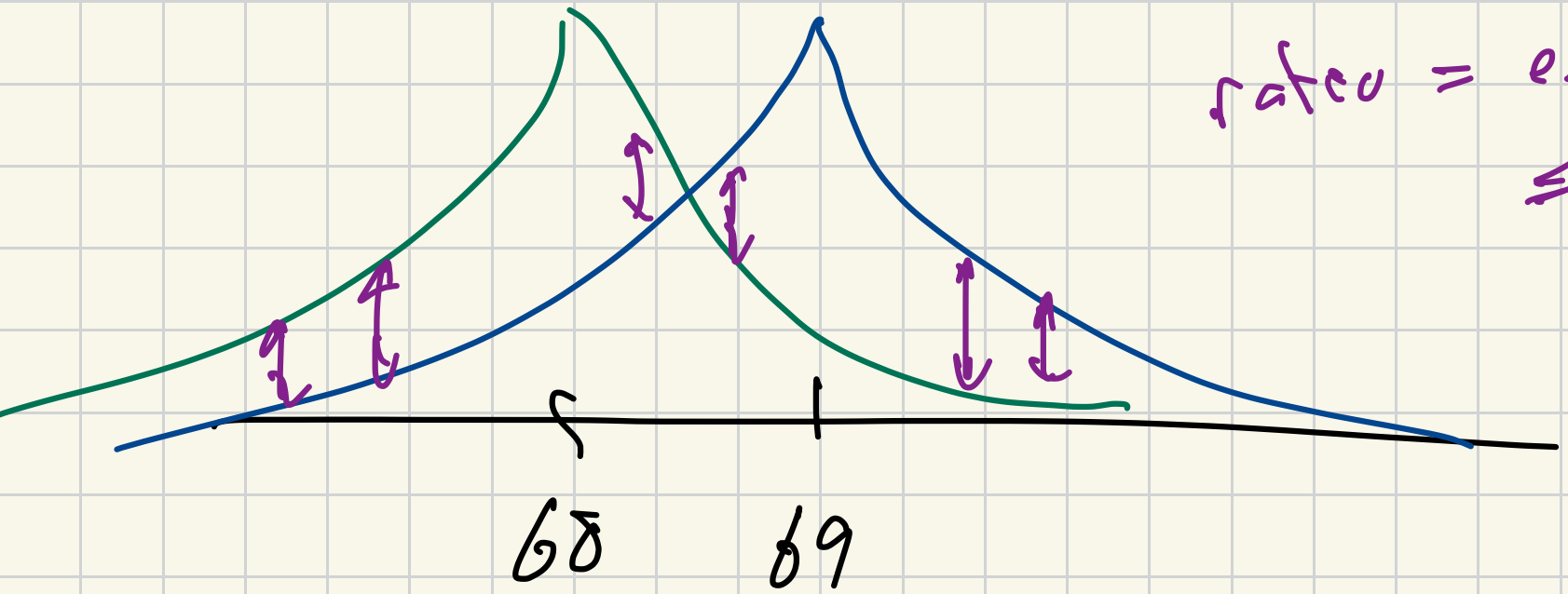
$x_2 = 69$ inches

$\text{diff}(x_1, x_2) = 1$

$s = q(x_1) = x_1 + \text{Lap}(0, \omega)$ $\quad \omega = 1/\varepsilon$



$$\frac{\Pr[s_1 = 70]}{\Pr[s_2 = 70]} = \frac{\frac{1}{2\omega}\exp(|68-70|/\omega)}{\frac{1}{2\omega}\exp(|69-70|/\omega)} = \exp\left(\tfrac{1}{2}|68-70| - \tfrac{1}{2}|69-70|\right) = \exp\left(\tfrac{1}{\omega}\right) = \exp(\varepsilon)$$



$$\text{ratio} = \exp(\epsilon)$$
$$\approx 1+\epsilon$$

Binary Example $X_1 = 1$ $X_2 = 0$

$$q(x_1) = x_1 + Lap(1/\varepsilon)$$

$$\frac{Pr[q(x_1) = 1.2]}{Pr[q(x_2) = 1.2]} = \frac{\exp(|1.2-1|/(1/\varepsilon))}{\exp(|1.2-0|/(1/\varepsilon))}$$

$$= \exp(\varepsilon(|1.2-1| - |1.2-0|))$$

$$= \exp(\varepsilon)$$

Database:

$X_1 = [1, 0, 1, 0, 0, 0]$

$X_2 = [1, 0, 0, 0, 0, 0]$

↑ mg data

$q_R(x_i) = |R \cap X_i| + Lap(1/\varepsilon)$

↑ range