L14: Streaming : Frequent Items and Quantiles

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Data is Big? Too big to fit on computer or in memory

1. Parallelism More computers

MapReduce

2. Sampling very large X data

$S \sim X$ $S = \{s_1, s_2, \ldots s_n\}$

$n << |X|$

3. Streaming $X = \langle x_1, x_2, x_3, \ldots x_i, \ldots x_n \rangle$

Read each x_i once, but not store

Maintain small space summary.

Data $\quad A = \langle a_1, a_2, a_3, \ldots a_i], \ldots a_n \rangle$

ex. $a_i \in \mathbb{R}$ $\qquad A_i = \langle a_1, a_2, \ldots a_i \rangle$

$a_1 \; a_2 \; \ldots \; a_i \quad a_{i+1}$

store $\boxed{\begin{matrix} V_i \\ i \end{matrix}} \Rightarrow V_{i+1}$

$$\frac{(V_i \cdot i) + a_{i+1}}{i+1}$$

mean (A_i)

$$\frac{1}{i} \sum_{j=1}^{i} a_i = V_i$$

sum $\boxed{\begin{matrix} S_i = \sum_{j=1}^{i} a_i \\ i = \# \text{ steps} \end{matrix}}$ $\oplus a_{i+1} \rightarrow$ $\boxed{\begin{matrix} S_{i+1} = S_i + a_{i+1} \\ i+1 = i+1 \end{matrix}}$

$$V_i = \frac{S_i}{i}$$

$$\frac{1}{i} \sum_{j=1}^{i} (a_j - E[A_i])^2 = \mathrm{Var}[A_i]$$

$$V_{i+1} = \frac{S_{i+1}}{i+1}$$

Variance $[A_i]$

$$\frac{1}{i}\sum_{j=1}^{i}\left(a_i - E[A_i]\right)^2 = \frac{1}{i}\sum_{j=1}^{i} a_i^2 - \left(\frac{1}{i}\sum_{j=1}^{i} a_i\right)^2$$

$$= E[a_i^2] - \left(E[a_i]\right)^2$$

store

$$\boxed{\begin{array}{c} i \\ S_i \\ Q_i \end{array}}$$

$$S_i = \sum_{j=1}^{i} a_j$$

$$Q_i = \sum_{j=1}^{i} (a_j)^2$$

$$\mathrm{Var}[A_i] = \frac{Q_i}{i} - \left(\frac{S_i}{i}\right)^2$$

Resevoir Sampling

$\langle a_1, a_2, \dots a_n \rangle$ stream

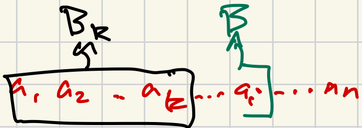
Goal: Maintain Random Sample $B \subset A$
without replacement $|B| = k$

1. Keep first k item $B = \{a_1, a_2, \dots a_k\}$
2. for $j = k+1 \dots$ to n

   keep a_j in B w/p $\frac{k}{j}$

   ↳ boot a random $b \in B$ from B.

   o.w. keep B the same



Stream $A = \langle a_1, a_2, \ldots a_i \ldots a_n \rangle$

$a_i \in [m]$

$[m] = $ IP addresses

$[m] = $ k-grams

both m and n too big.

label $j \in [m]$ $\log m$ bits

counter $i \in [n]$ $\log n$ bits

approx freq

$\hat{f}_j$

$|f_j - \hat{f}_j| \leq \varepsilon n$

frequency $f_j = |\{a_i \in A \mid a_i = j\}|$
$j \in [m]$

Heavy-Hitter j s.t. $f_j > \phi n$

$\phi = 0.1$
$\varepsilon = 0.01$

ok $f_j > \phi n - \varepsilon n$

MAJORITY

$A = (a_1 \ldots a_n)$

$a_i \in [m]$

if (some $f_j > \frac{m}{2}$) $\rightarrow$ output j

else output anything $j \in [m]$

[counter $\in [n]$ | label $j \in [m]$

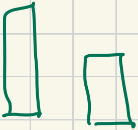
c $\qquad$ l

if ($a_i = l$) $\quad c = c + 1$

else $\quad c = c - 1$

if ($c < 0$)

$l = a_i$; $c = 1$



l_1 $\quad$ l_2

Majority

Majority(A)

Set $c = 0$ and $\ell = \emptyset$
for $i = 1$ **to** $\cancel{m}\ n$ **do**
  if $(a_i = \ell)$ **then**
    $c = c + 1$
  else
    $c = c - 1$
  if $(c < 0)$ **then**
    $c = 1,\ \ell = a_i$
return ℓ

Frequency Approximation — Misra-Gries

$k-1$ counters, $k-1$ labels

$\forall_{j \in [n]}$ $\hat{f}_j$ s.t. $f_j - \frac{n}{k} \le \boxed{\hat{f}_j} \le f_j$ apx freq. $k = \frac{1}{\varepsilon}$

$\boxed{k = \frac{1}{\varepsilon} \quad \frac{n}{k} = \varepsilon n}$

Labels $l_1, l_2, \ldots l_{k-1} = L$

counters $c_1, c_2, \ldots c_{k-1}$

for $(a_i \in A)$
  if $(a_i \in L)$ $a_i = l_j$
    $c_j = c_j + 1$
  else $(a_i \notin L)$
    if (some $c_j \le 0$) $\to$ else { decrease all counter by 1 $\forall_j$ $c_j = c_j - 1$
      $l_j = a_i$
      $c_j = 1$

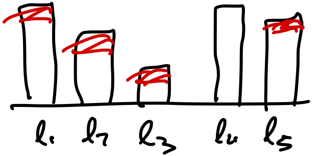
no labels unused.

Misra-Gries

counter array C : $C[1], C[2], \ldots, C[k-1]$
location array L : $L[1], L[2], \ldots, L[k-1]$

<u>Misra-Gries(A)</u>

  Set all $C[i] = 0$ and all $L[i] = \emptyset$
  for $i = 1$ **to** $\cancel{m}\ n$ **do**
    if $(a_i = L[j])$ **then**
      $C[j] = C[j] + 1$
    else
      if (some $C[j] = 0$) **then**
        Set $L[j] = a_i$ & $C[j] = 1$
      else
        for $j \in [k-1]$ **do** $C[j] = C[j] - 1$
  return C, L



how many total decrements?

$\hookrightarrow \dfrac{n}{k}$

Frugal Median

Frugal Median(A)

Set $\ell = 0$.
for $i = 1$ **to** m **do**
  if $(a_i > \ell)$ **then**
    $\ell \leftarrow \ell + 1$.
  if $(a_i < \ell)$ **then**
    $\ell \leftarrow \ell - 1$.
return ℓ.

Frugal Quantile

Frugal Quantile(A, ϕ) e.g. $\phi = 0.75$

Set $\ell = 0$.
for $i = 1$ **to** m **do**
  $r = \mathrm{Unif}(0, 1)$ (at random)
  if ($a_i > \ell$ **and** $r > 1 - \phi$) **then**
    $\ell \leftarrow \ell + 1$.
  if ($a_i < \ell$ **and** $r > \phi$) **then**
    $\ell \leftarrow \ell - 1$.
return ℓ.

Frequent Itemsets : Apriori

$$
\begin{aligned}
T_1 &= \{1, 2, 3, 4, 5\} \\
T_2 &= \{2, 6, 7, 9\} \\
T_3 &= \{1, 3, 5, 6\} \\
T_4 &= \{2, 6, 9\} \\
T_5 &= \{7, 8\} \\
T_6 &= \{1, 2, 6\} \\
T_7 &= \{0, 3, 5, 6\} \\
T_8 &= \{0, 2, 4\} \\
T_9 &= \{2, 4\} \\
T_{10} &= \{6, 7, 9\} \\
T_{11} &= \{3, 6, 9\} \\
T_{12} &= \{6, 7, 8\}
\end{aligned}
$$