Outlier Robust ICP Minimizing Fractional RMSD

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Distance Functions

$$\mathrm{RMSD}(D, M, T, \mu) = \sqrt{\frac{1}{|D|} \sum_{p \in D} ||T(p) - \mu(p)||^2}$$

Problem 1 [minimize rmsd]:
Align data point set D to model point set M under a set of transformations $\mathcal{T}$ to minimize

$$\min_{\substack{T \in \mathcal{T} \\ \mu : D \to M}} \mathrm{RMSD}(D, M, T, \mu)$$

- $\mathcal{T}$ = rotations, translations, scale, ...
- μ = matchings from D to M
- hard to optimize over both $\mathcal{T}$ and μ
- susceptible to outliers

$$\mathrm{FRMSD}(D, M, f, T, \mu) = \frac{1}{f^\lambda} \sqrt{\frac{1}{|D_f|} \sum_{p \in D_f} ||T(p) - \mu(p)||^2}$$

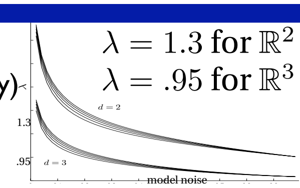
Let D_f be $f|D|$ points $p \in D$ with smallest residuals $||p - \mu(p)||$.

Problem 2 [minimize frmsd]:
Align data point set D to model point set M under a set of transformations $\mathcal{T}$ and fractions to minimize

$$\min_{\substack{T \in \mathcal{T} \\ \mu : D \to M \\ f \in [0,1]}} \mathrm{FRMSD}(D, M, f, T, \mu)$$

- minimum near true fraction of outliers

Optimal value of λ depends on
noise of model & fraction of inliers (weakly)
so that aligned points are more likely inliers than outliers



λ	time (s)	# iter.	RMSD	FRMSD	f
1	0.142	10.38	0.158	0.225	0.701
1.3	0.069	3.81	0.170	0.248	0.749
2	0.059	3.06	0.170	0.303	0.750
3	0.061	3.17	0.170	0.404	0.750
4	0.062	3.21	0.171	0.538	0.751
5	0.063	3.30	0.172	0.717	0.751

- FRMSD is robust for $\lambda \in [1, 5]$
- FICP has larger radius of convergence with $\lambda = 3$

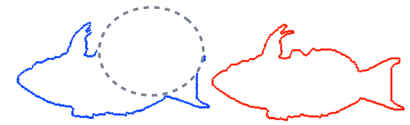
Motivation: Registration with Outliers

Registration is often skewed by outliers

Outlier detection depends on registration

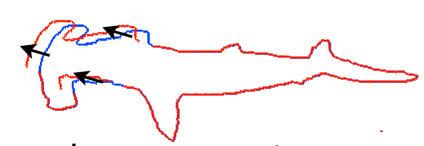
We register point sets and find outliers in one algorithm

Occlusion



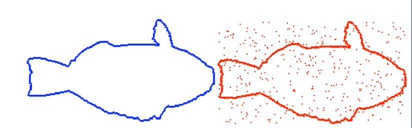
- partial matches
- scans from different view
- data set has grown

Deformation



- changes over time
- comparison of similar objects

New Data



- measurement error
- spurious/unrelated data

Fractional ICP

repeat

1. Compute closest $f|D|$ points: D_f
2. Compute transformation: $T_i \in \mathcal{T}$
   to minimize $\mathrm{RMSD}(D_f, M, T_i, \mu_{i-1})$
3. Compute matching: $\mu_i \in D \to M$
   to minimize $\mathrm{RMSD}(D, M, T_i, \mu_i)$
4. Compute fraction: $f_i \in [0, 1]$
   to minimize $\mathrm{FRMSD}(D, M, f_i, T_i, \mu_i)$

until $(\mu_i = \mu_{i-1}$ and $f_i = f_{i-1})$

Theorem: *Fractional ICP aligning D and M always converges to a local minimum of FRMSD in the space of all transformations $\mathcal{T}$, matchings $\{D \to M\}$, and fractions of inliers $[0, 1]$.*

Proof Sketch: The state (f, T, μ) only changes at steps 2, 3, and 4. At each step $\mathrm{FRMSD}(D, M, f, T, \mu)$ cannot increase.

- Fixing a distance threshold for μ may not converge.
- Fixing a fraction f (TrICP) does not find a local minimum.

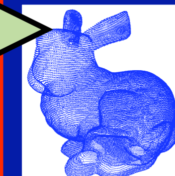
Funnel of convergence is larger than TrICP, but smaller than ICP. ICP has smaller parameter space.

Algo.	λ	rotation 5°	10°	25°	50°
ICP	-	0.999	0.997	0.994	0.962
TrICP	3	0.875	0.870	0.853	0.816
FICP	3	0.952	0.945	0.909	0.875
FICP	1.3	0.857	0.473	0.141	0.060

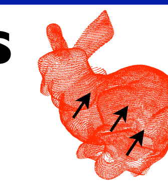
- TrICP heuristically searches for f outside of loop.

Experiments

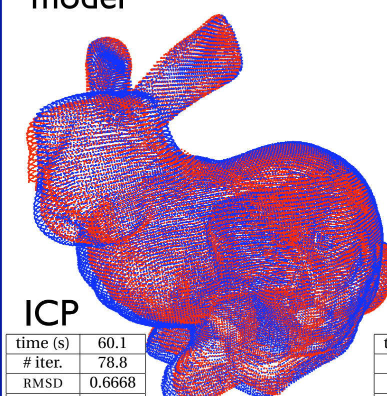
Stanford bunny:
25% deformation
5° rotation



model

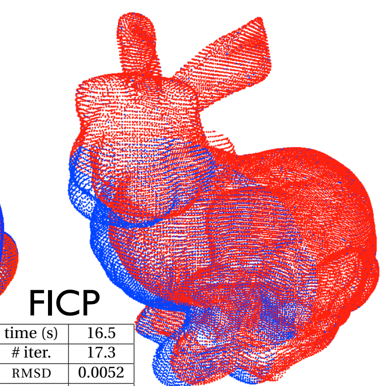


data



ICP

time (s)	60.1
# iter.	78.8
RMSD	0.6668
FRMSD	0.6668
f	1.0



FICP

time (s)	16.5
# iter.	17.3
RMSD	0.0052
FRMSD	0.0124
f	0.750

Stanford dragon: scans at 0° and 48°