Little Tricky Logic:
Misconceptions in the Understanding of LTL

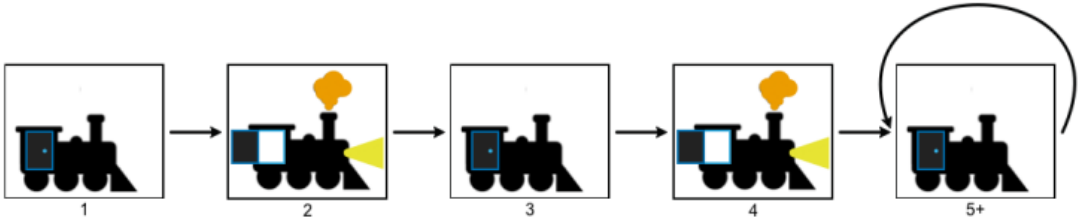
Ben Greenman
Sam Saarinen
Tim Nelson
Shriram Krishnamurthi

@ Brown University

‹Programming› 2023

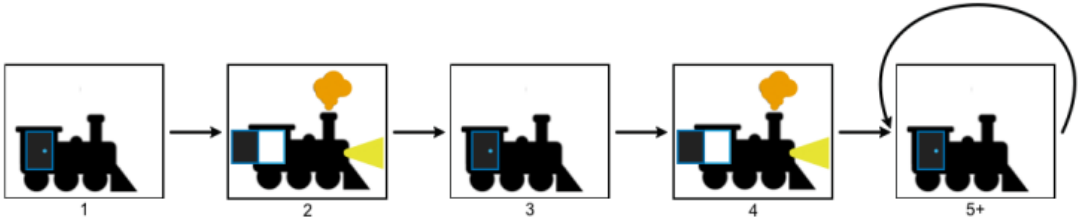
LTL = **L**inear **T**emporal **L**ogic

LTL = **L**inear **T**emporal **L**ogic



For systems that change over time

LTL = **L**inear **T**emporal **L**ogic



For systems that change over time

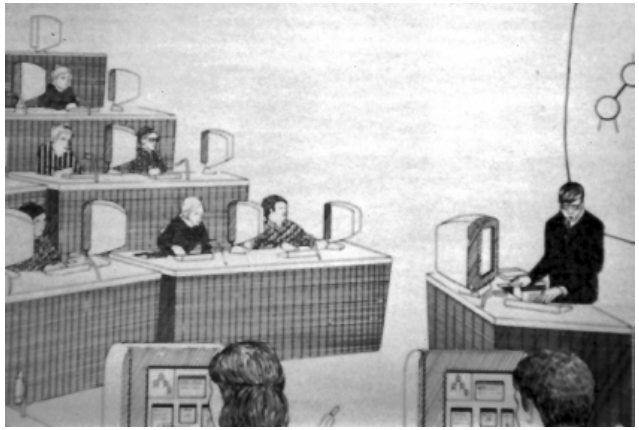
- ✓ Expressive
- ✓ Supports good decision procedures
- ✓ Small

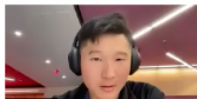
.... and easy to learn?

RQ. In **what ways** is LTL tricky, and **what can we do** about it?

RQ. In **what ways** is LTL tricky, and **what can we do** about it?

2 years of studies with researchers and students
4 survey rounds





qualtrics

Quiz Time!

Quiz Format:

one question,
possible answers,
you decide yes/no

✓ ✗

Question

✓ Possible Answer 1

✗ Possible Answer 2

✓ Possible Answer 3

LTL Operators:

`always`	`(G)`
`eventually`	`(F)`
`after`	`(X)`
`until`	`(U)`

Part 1:

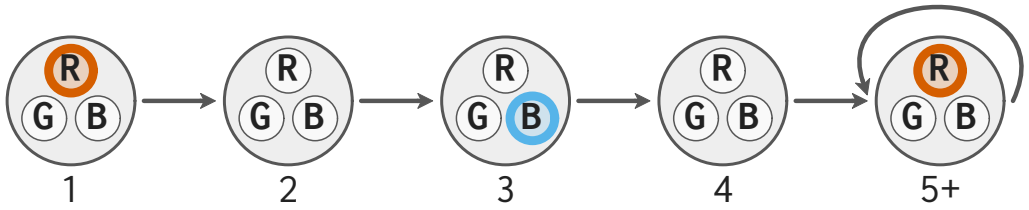
Formulas vs. Traces

Q. Do the traces below satisfy this formula?

`{eventually Red} and {eventually Green}`

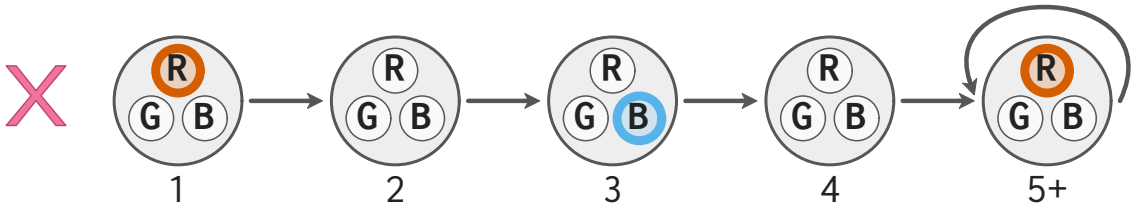
Q. Do the traces below satisfy this formula?

{eventually Red} and {eventually Green}



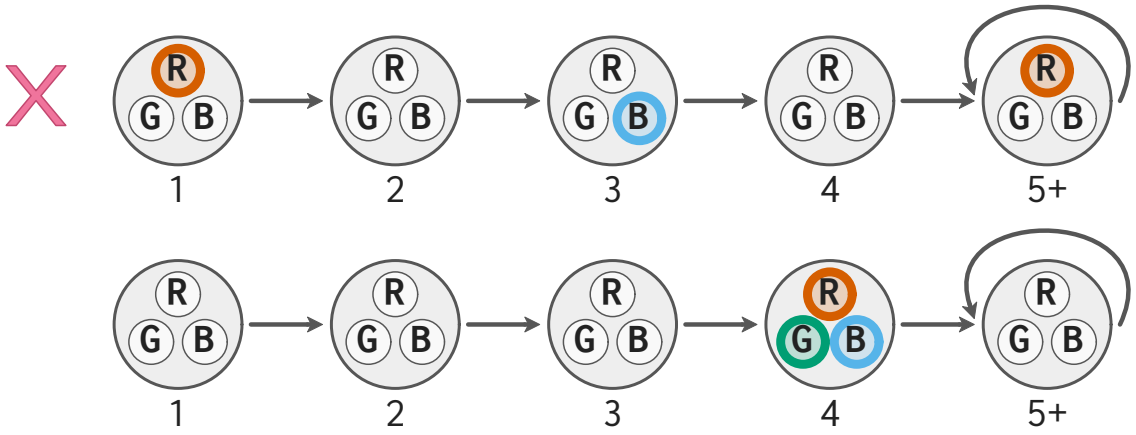
Q. Do the traces below satisfy this formula?

{eventually Red} and {eventually Green}



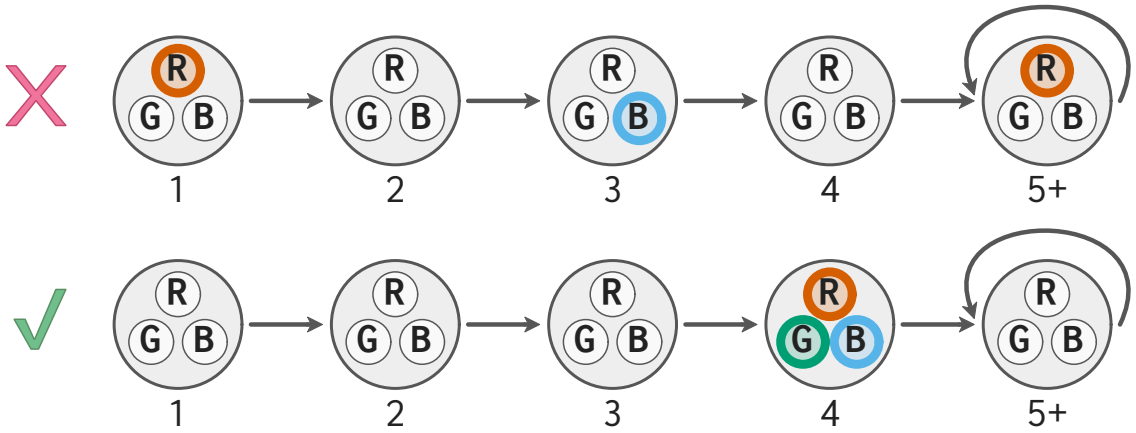
Q. Do the traces below satisfy this formula?

{eventually Red} and {eventually Green}



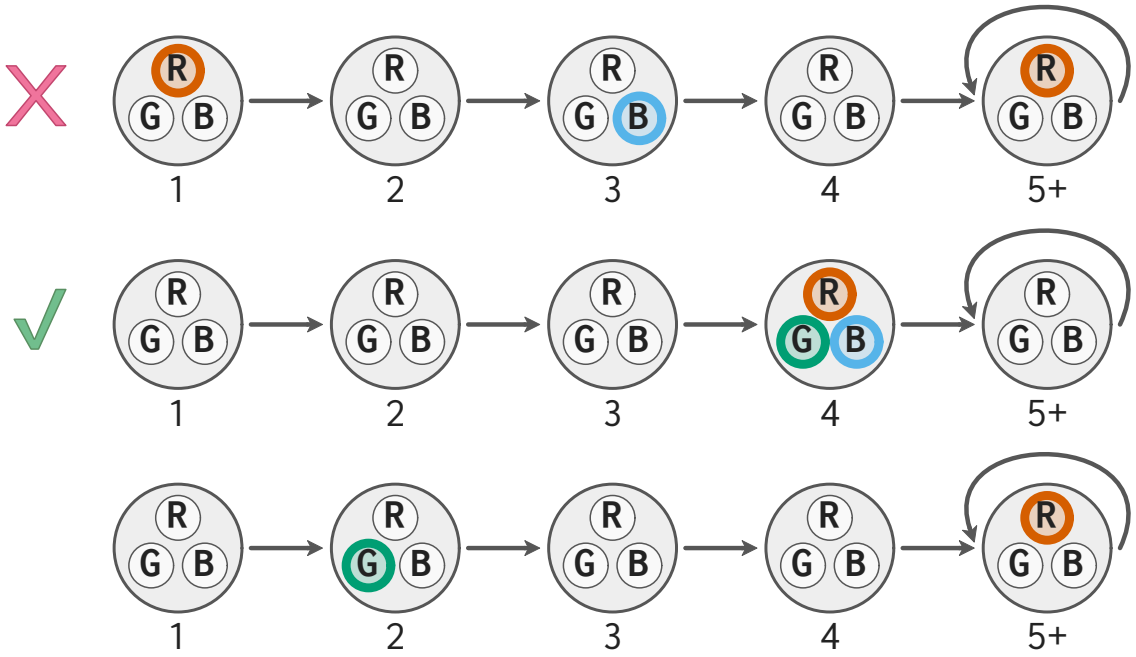
Q. Do the traces below satisfy this formula?

{eventually Red} and {eventually Green}



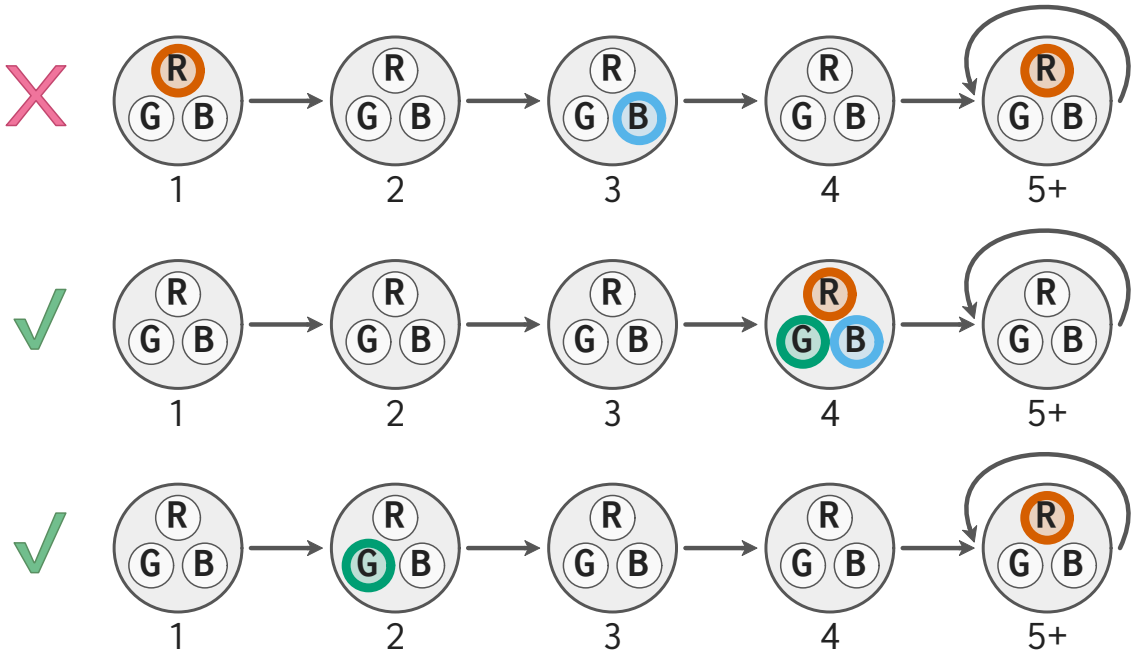
Q. Do the traces below satisfy this formula?

{eventually Red} and {eventually Green}



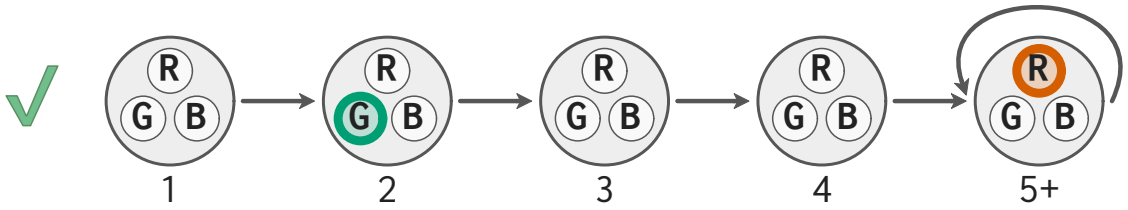
Q. Do the traces below satisfy this formula?

{eventually Red} and {eventually Green}



Q. Do the traces below satisfy this formula?

{eventually Red} and {eventually Green}

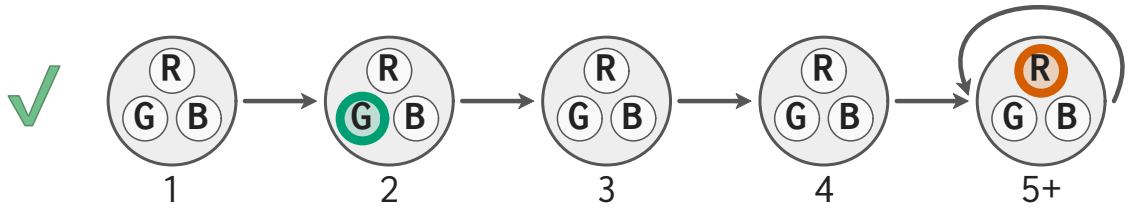


Q. Do the traces below satisfy this formula?

`{eventually Red} and {eventually Green}`

Not satisfied, because Green comes before Red

`Bad Prop` misconception

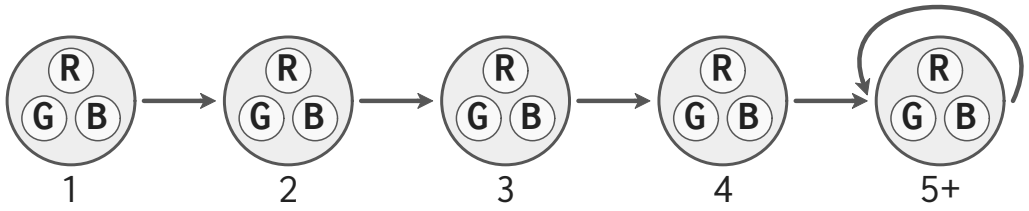


Q. Do the traces below satisfy this formula?

Red

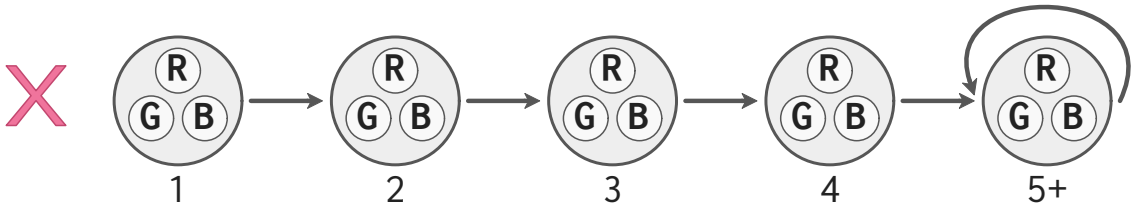
Q. Do the traces below satisfy this formula?

Red



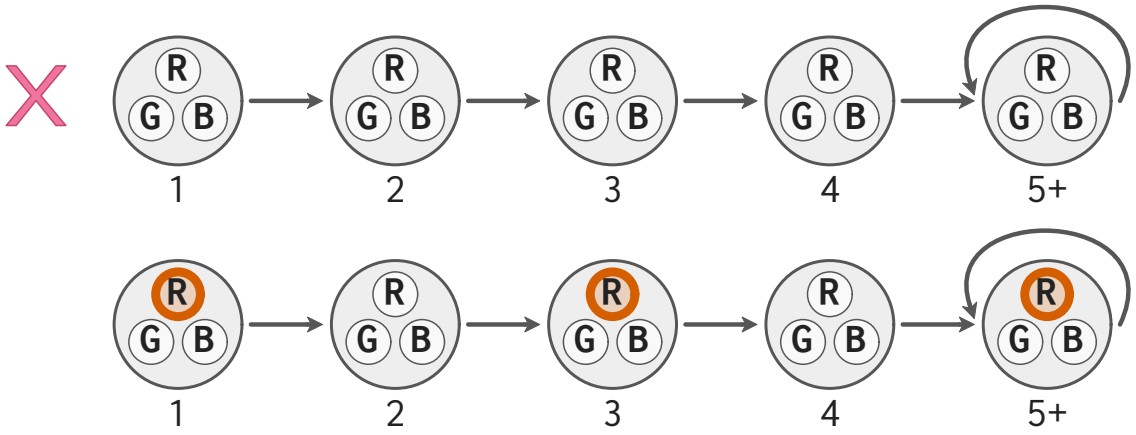
Q. Do the traces below satisfy this formula?

Red



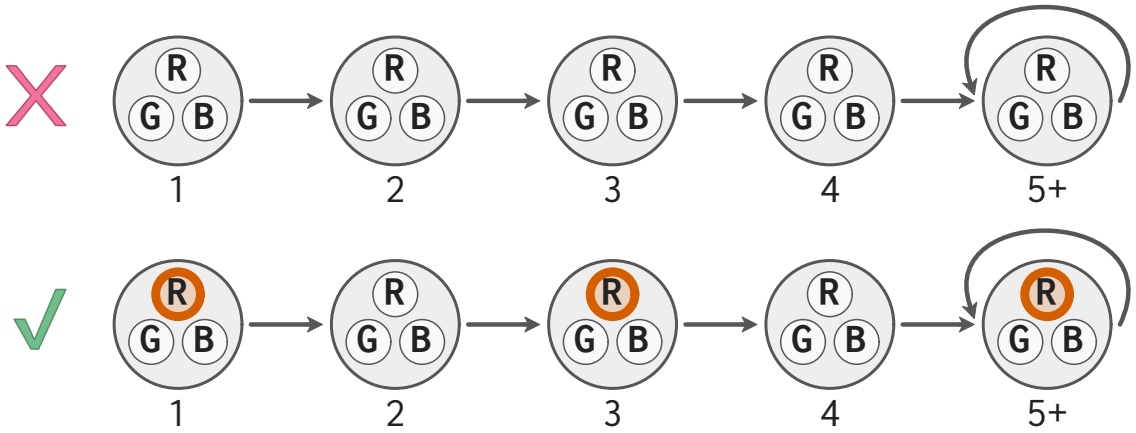
Q. Do the traces below satisfy this formula?

Red



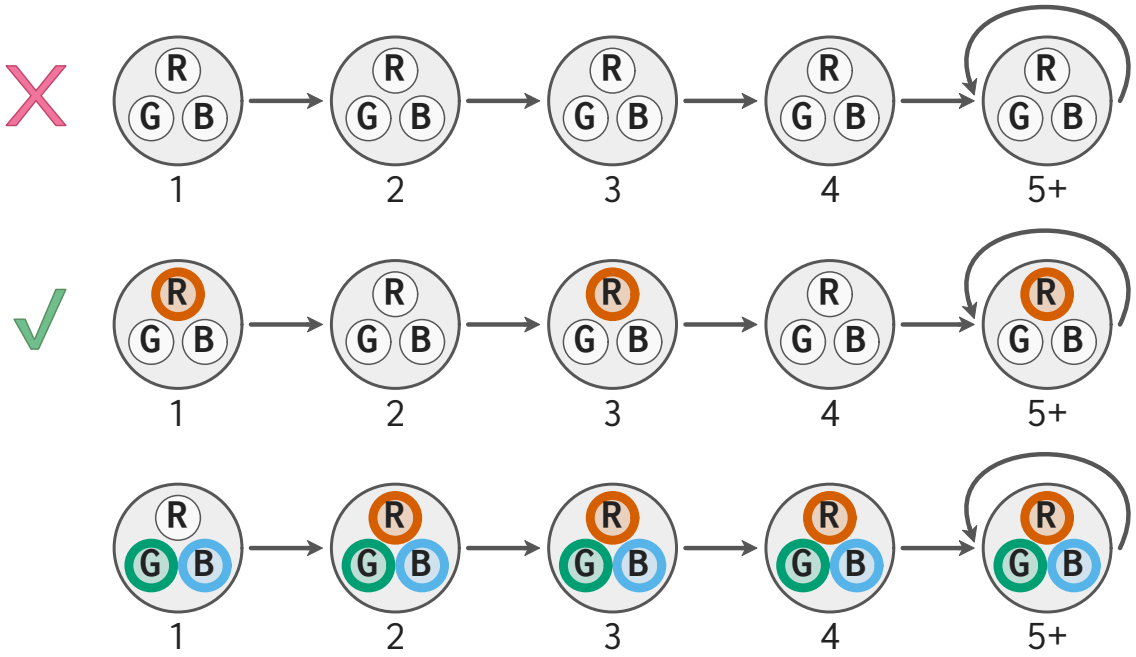
Q. Do the traces below satisfy this formula?

Red



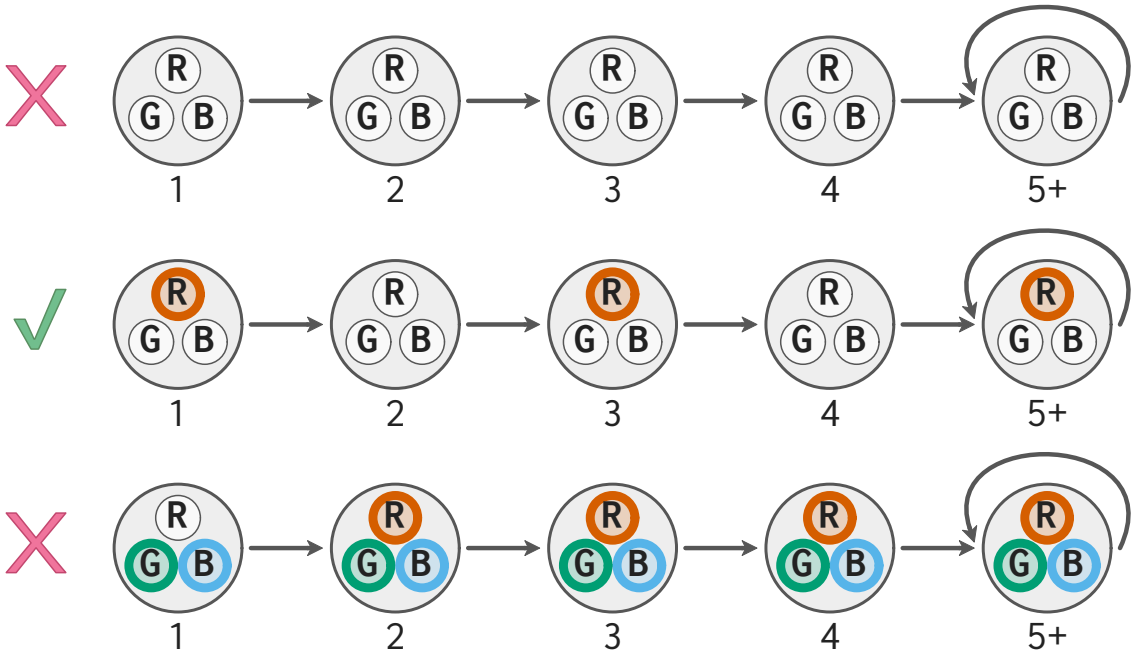
Q. Do the traces below satisfy this formula?

Red



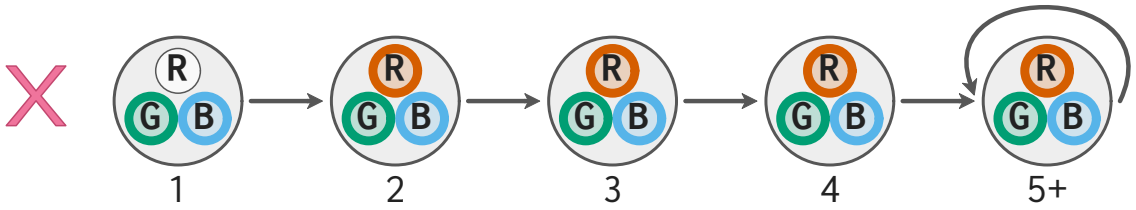
Q. Do the traces below satisfy this formula?

Red



Q. Do the traces below satisfy this formula?

Red

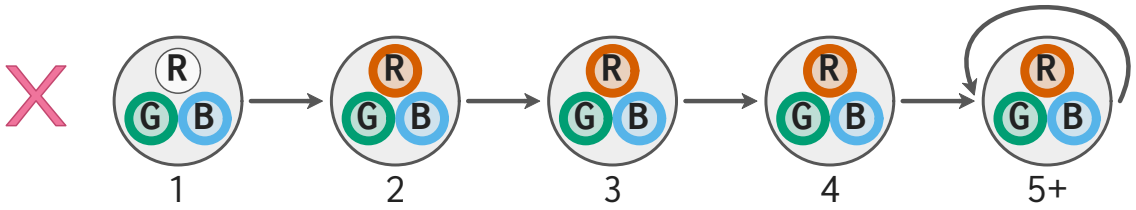


Q. Do the traces below satisfy this formula?

Red

Satisfied because Red is on at some point

`Implicit F` misconception

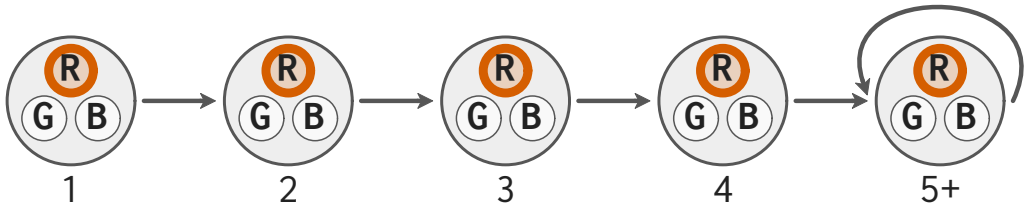


Q. Do the traces below satisfy this formula?

`Red until Blue` [strong until]

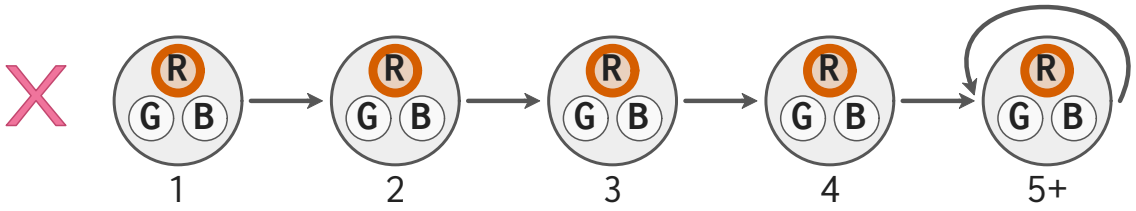
Q. Do the traces below satisfy this formula?

`Red until Blue` [strong until]



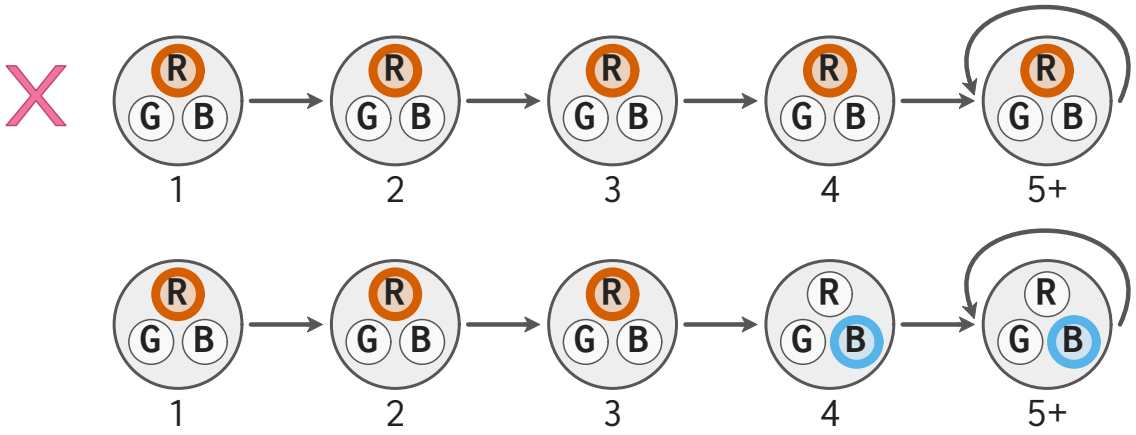
Q. Do the traces below satisfy this formula?

`Red until Blue` [strong until]



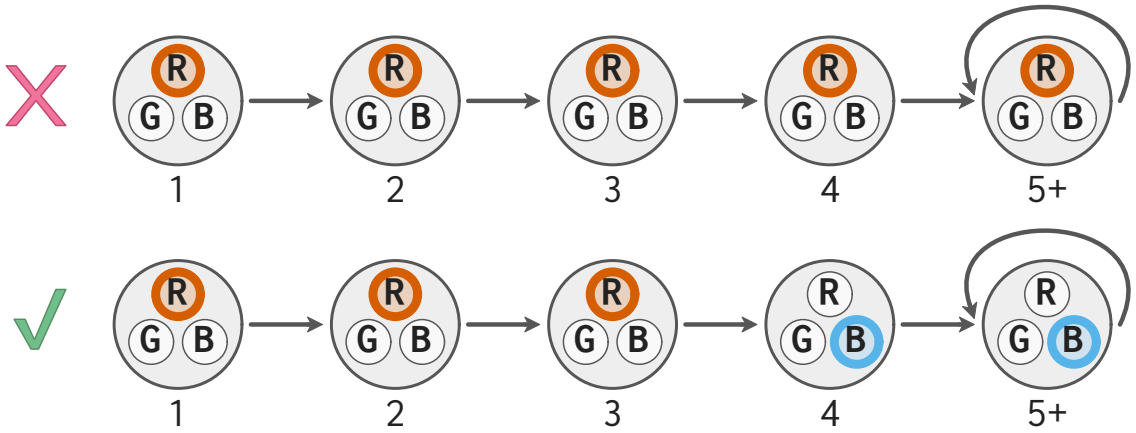
Q. Do the traces below satisfy this formula?

`Red until Blue` [strong until]



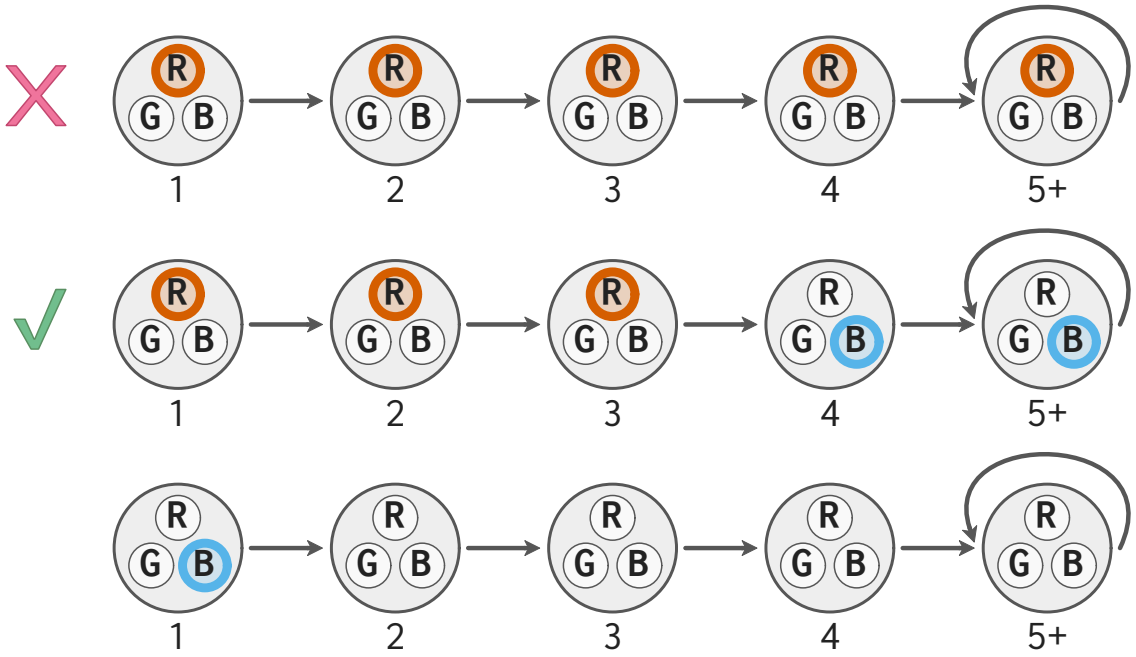
Q. Do the traces below satisfy this formula?

`Red until Blue` [strong until]



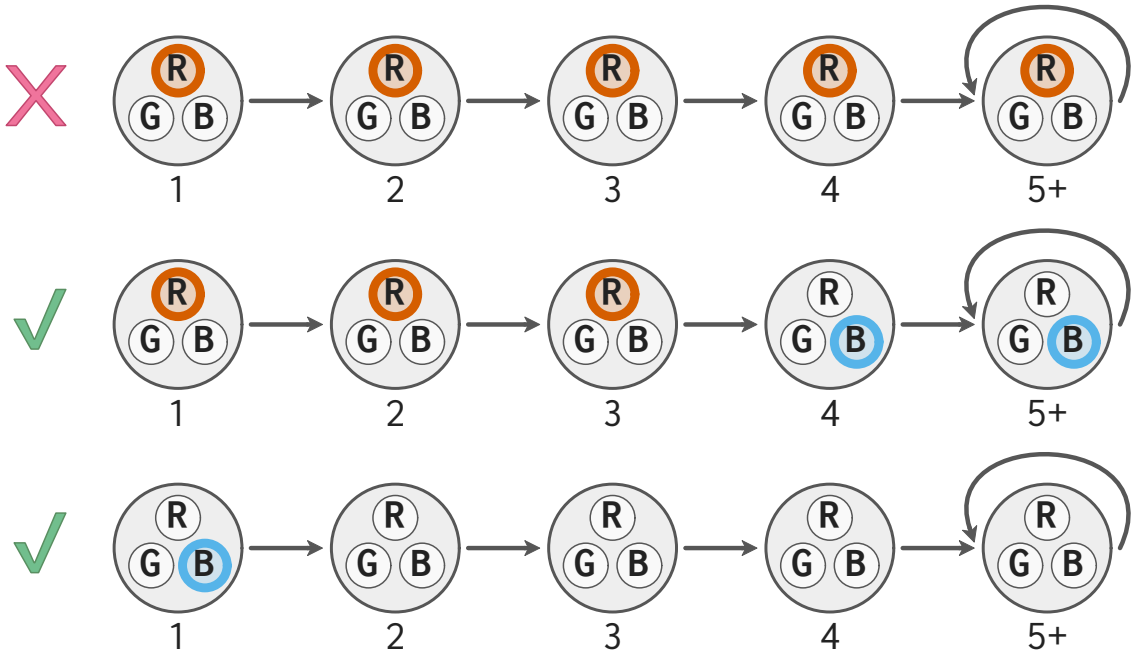
Q. Do the traces below satisfy this formula?

`Red until Blue` [strong until]



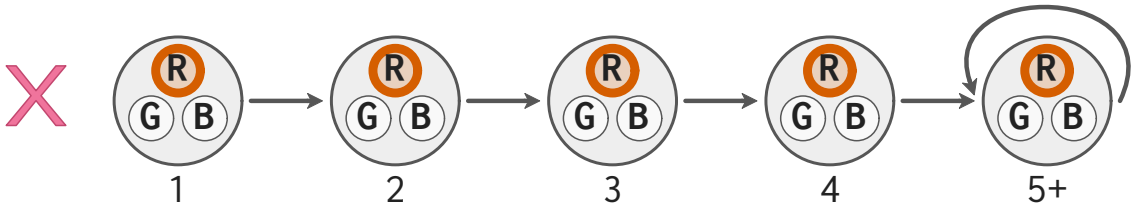
Q. Do the traces below satisfy this formula?

`Red until Blue` [strong until]



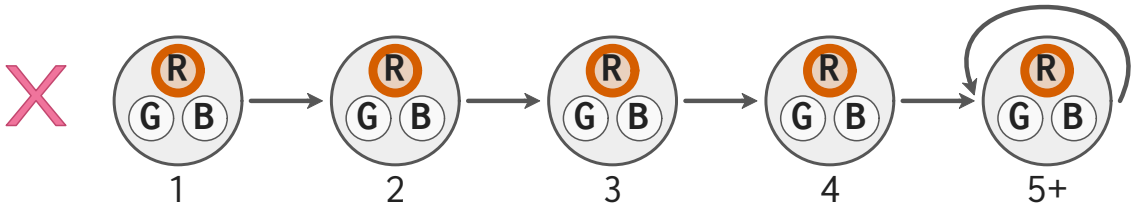
Q. Do the traces below satisfy this formula?

`Red until Blue` [strong until]



Q. Do the traces below satisfy this formula?

`Red until Blue` [strong until]



Satisfied because Blue may stay off

Even among researchers!

`Weak U` misconception

Part 2:

LTL to English

Q. Translate to English

{Red until Blue} and {always Red}

Q. Translate to English

{Red until Blue} and {always Red}

"Red is always on"

Q. Translate to English

{Red until Blue} and {always Red}

"Red is always on"

Q. Translate to English

{Red until Blue} and {always Red}

"Red is always on"

"Red is always on and Blue is eventually on"

Q. Translate to English

{Red until Blue} and {always Red}

✗ "Red is always on"

✓ "Red is always on and Blue is eventually on"

Q. Translate to English

{Red until Blue} and {always Red}

✗ "Red is always on"

✓ "Red is always on and Blue is eventually on"

"This statement can never be satisfied"

Q. Translate to English

{Red until Blue} and {always Red}

✗ "Red is always on"

✓ "Red is always on and Blue is eventually on"

✗ "This statement can never be satisfied"

Q. Translate to English

{Red until Blue} and {always Red}

"This statement can never be satisfied"

Q. Translate to English

{Red until Blue} and {always Red}

When Blue turns on, Red **must** be off

`Exclusive U` misconception

"This statement can never be satisfied"

Q. Translate to English

{eventually Red} implies {always Blue}

Q. Translate to English

`{eventually Red} implies {always Blue}`

"if Red is ever on, then Blue is always on"

Q. Translate to English

`{eventually Red} implies {always Blue}`

✓ "if Red is ever on, then Blue is always on"

Q. Translate to English

`{eventually Red} implies {always Blue}`

✓ "if Red is ever on, then Blue is always on"

"Red is on at some point, after which Blue is on"

Q. Translate to English

`{eventually Red} implies {always Blue}`

✓ "if Red is ever on, then Blue is always on"

✗ "Red is on at some point, after which Blue is on"

Q. Translate to English

`{eventually Red} implies {always Blue}`

✗ "Red is on at some point, after which Blue is on"

Q. Translate to English

`{eventually Red} implies {always Blue}`

Red **will** turn on (also, a precedence issue)

`Bad Prop` misconception

"Red is on at some point, after which Blue is on"

Part 3:

English to LTL

Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state

Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state

```
Impossible!
```

Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state

`Impossible!`

Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state

```
Impossible!
```

```
{eventually Red} and {always {Red => always !Red}}
```

Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state

✗ `Impossible!`

✗ `{eventually Red} and {always {Red => always !Red}}`

Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state

✗ `Impossible!`

✗ `{eventually Red} and {always {Red => always !Red}}`

`{eventually Red} and {always {Red => after {always !Red}}}`

Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state

✗ `Impossible!`

✗ `{eventually Red} and {always {Red => always !Red}}`

✓ `{eventually Red} and {always {Red => after {always !Red}}}`

Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state

```
{eventually Red} and {always {Red => always !Red}}
```

Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state

An implication constrains the **next state**

`Bad State Index` misconception

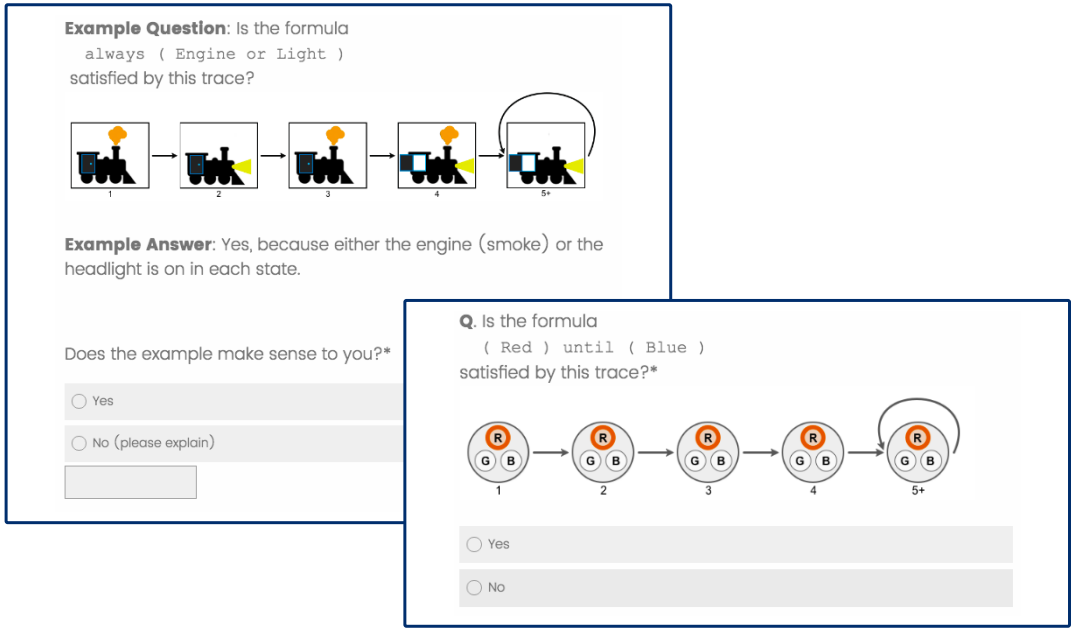
```
{eventually Red} and {always {Red => always !Red}}
```

All Done!

Simple formulas, yet
subtle issues and **expert blind spots**

Quiz Q's Based on 3 Instruments

- Trace Satisfaction
- LTL to English
- English to LTL



cs.brown.edu/~bgreenma/ltl-instruments.pdf

Code Book for Analysis

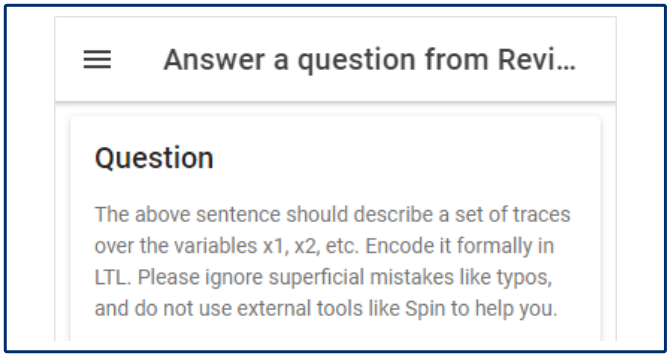
Bad Prop	Implicit F
Bad State Index	Implicit G
Bad State Quantification	Other Implicit
Exclusive U	Weak U

Coding Rubric in paper, past versions in artifact

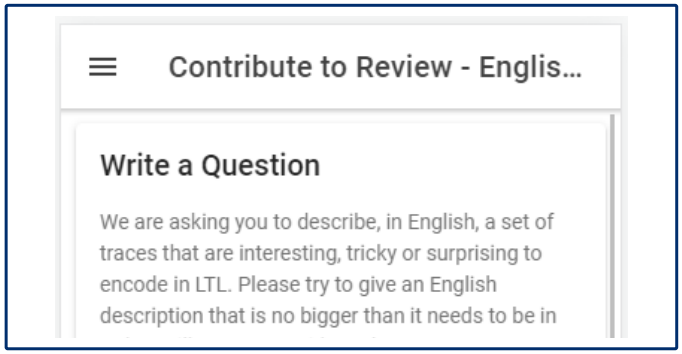
Software: Quizius

Class-sourcing to discover misconceptions

1. Answer Top Q's



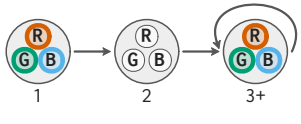
2. Submit New Q's



What Next?

What Next?

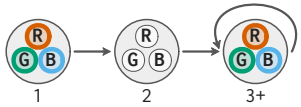
1. Teach Better



our instruments can help!

What Next?

1. Teach Better



our instruments can help!

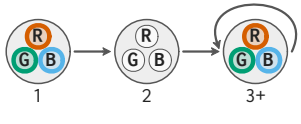
... but learners are everywhere
not just in classrooms

amazon Meta

NETFLIX

What Next?

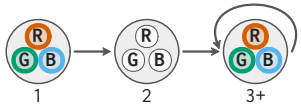
1. Teach Better



our instruments can help!

What Next?

1. Teach Better



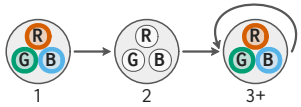
our instruments can help!

2. Build Tools

guard against misconceptions

What Next?

1. Teach Better



our instruments can help!

2. Build Tools

guard against misconceptions

3. Design Logics

Alloy 6 **Electrum**

our findings have inspired changes

Thank You!

benjamin.l.greenman@gmail.com