5-Axis Freeform Surface Milling using Piecewise Ruled Surface Approximation *

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Abstract

This paper presents a 5-axis side milling scheme for freeform surfaces based on automatic piecewise ruled surface approximation.

With this scheme, resulting surface finish is accurate and pleasing, and has a smaller scallop height compared to ball-end milling. The ruled surface approximation can be made arbitrarily precise resulting in an overall fast milling operation that satisfies tight tolerances, and smoother surface finish.

The class of surfaces that can take advantage of this type of 5-axis milling operation includes both convex and saddle-like (hyperbolic) shapes.

1 Introduction

The automatic toolpath generation for manufacturing of models consisting of freeform surfaces is a difficult problem addressed by numerous researchers [Bob85, Choi89, Choi93, Chou89, Elb93, Elb94, Fers86, Jera91, Lone87, Sait91, Spen90, Zhan86].

3-axis machining [Bob85, Choi89, Fers86, Jera91, Lone87, Sait91, Spen90, Zhan86] is more frequently used than 4- or 5-axis machining modes. While deriving a multi

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(i.e. 4- or 5-) axis toolpath is a relatively simple task, resolving the *tool accessibility* question can be difficult. Finding out whether the tool can access the surface without gouging into it, is significantly more difficult when a tool can be arbitrarily oriented. We differentiate between *local accessibility* and *global accessibility*. The former can be resolved by inspecting only the neighborhood region of the surface. The latter must take into consideration all other surfaces, or objects that can interfere with the tool or its holder. Clearly, the local tool accessibility is an easier problem. In this paper, we do not strive to solve the global tool accessibility problem, and only show how the presented milling approach can guarantee local access.

The larger the machining tool, the smaller the resulting scallop height, or the excessive material not removed, that might result for the same toolpath. Moreover, larger tools reduce machining times. A smooth finish usually results by using flat end tools (see Plates 1 and 2). Machining using ball-end tools is slow because of a vanishing cutting speed at the tip of the tool; an impediment that shows up neither in the 5-axis flat end milling mode nor in the 5-axis side milling approach proposed herein.

Limited research has been conducted toward the use of 4- and 5-axis freeform surface machining. In [Chou89], an attempt is made using local surface analysis to compute 5-axis flat-end toolpaths for freeform surfaces that are generally convex but with isolated non-convex regions. No discussion is made as to how the convex regions can be detected and isolated. Only local accessibility is considered and resolved numerically. In [Elb1], a robust optimization method is derived, based on global symbolic shape analysis, to automatically detect and differentiate the surface regions that can be machined using a flat end tool in a 5-axis mode and the surface regions that must be machined using a ball end tool in a 3-axis mode. In [Elb93], a scheme for the generation of an almost optimal toolpath for freeform surfaces is derived based on iso-parametric curves. By utilizing iso-parametric curves, the resulting toolpath is exact and compact as opposed to contoured piecewise linear data that is generally used. By extracting iso-parametric curves and adaptively eliminating redundancies in the iso-parametric curves' toolpath coverage, the length of the toolpath is optimized. This scheme of adaptive iso-parametric curves can also be employed for 5-axis flat-end milling, as is done for example in [Elb94].

Tool accessibility, in 5-axis machining modes, is a most difficult and challenging problem. In [Chen89], an attempt is made to classify the directions from which a model is accessible by deriving the Gauss map, $\mathcal{G}$ [DoCa76], and the visibility map, $\mathcal{V}$, of the model. While $\mathcal{G}$ is a well known result from the field of differential geometry [DoCa76], $\mathcal{V}$ introduces the set of all directions from which a surface is completely visible. In [Elb2], this notion is extended to freeform surfaces. This classification solves only local accessibility and does not detect global interference. In [Elb94], a method is suggested that provides a solution for the global interference problem for machined surfaces that are convex. This method reduces the 5-axis global accessibility problem into a 3-axis global accessibility problem that is easier to resolve, by warping the space above the machined surface. The method in [Elb94], can always be utilized whenever an operation of 5-axis flat-end mill that is normal to the surface is feasible.

In this paper, we propose an approach to 5-axis machining that would extend the class of machinable freeform surfaces to include both convex and saddle-like (hyperbolic) surfaces. The proposed approach exploits the *side* of the tool to machine the surface, presenting an effective tool of zero curvature and hence smaller scallop height. The paper is organized as follows. Section 2 describes how a ruled surface approximation can be derived for an arbitrary piecewise polynomial or rational surface using the B-spline tensor product representation, following [Elb3]. Section 3 extends this surface approximation to formulate a side milling technique for convex and saddle-like surfaces.

Section 4 presents some tested results on several convex and saddle-like surfaces. Section 5 discusses conclusions and future work.

All presented results, including the tested machined parts, were created with the aid of Alpha_1, a NURBs based solid modeling system that is developed at the University of Utah.

2 Piecewise Ruled Surface Approximation

Let $S_1(u, v)$ and $S_2(r, s)$ be two parametric surfaces.

Definition 1 *The distance between point* $S_1(u_0, v_0)$ *and surface* $S_2(r, s)$ *is*

$$Dist(S_1(u_0, v_0), S_2(r, s)) = \min_{r,\, s} \|S_1(u_0, v_0) - S_2(r, s)\|. \tag{1}$$

Definition 2 *The distance between two surfaces* $S_1(u, v)$ *and* $S_2(r, s)$ *is*

$$Dist(S_1(u, v), S_2(r, s)) = \max_{u,\, v} \min_{r,\, s} \|S_1(u, v) - S_2(r, s)\|. \tag{2}$$

Let $S(u, v)$ be a polynomial B-spline surface. Let $C_1(u) = S(u, Vmin)$ and $C_2(u) = S(u, Vmax)$ be the $Vmin$ and $Vmax$ boundary curves of $S(u, v)$ respectively. Assume $C_1(u) \neq C_2(u)$. Let,

$$R(u, v) = vC_1(u) + (1 - v)C_2(u), \qquad 0 \leq v \leq 1, \tag{3}$$

be the ruled surface constructed between $C_1(u)$ and $C_2(u)$. Let $\hat{R}(u, v)$ be the representation for $R(u, v)$ in the same space as that of $S(u, v)$. That is, $\hat{R}$ and S share the same v order and continuity (knot vector). $\hat{R}(u, v)$ can be obtained from $R(u, v)$ via appropriate degree raising [Cohe86a, Cohe86b] and refinement [Boeh80, Cohe80] of the linear (ruled) direction, v. Then,

$$
\begin{aligned}
& \max_{u,\, v} \|S(u,v) - \hat{R}(u,v)\| \\
= & \max_{u,\, v} \|\sum_{i=0}^{m}\sum_{j=0}^{n} P_{ij} B_{i,\tau}^{m}(u) B_{j,\xi}^{n}(v) - \sum_{i=0}^{m}\sum_{j=0}^{n} Q_{ij} B_{i,\tau}^{m}(u) B_{j,\xi}^{n}(v)\| \\
= & \max_{u,\, v} \|\sum_{i=0}^{m}\sum_{j=0}^{n} (P_{ij} - Q_{ij}) B_{i,\tau}^{m}(u) B_{j,\xi}^{n}(v)\| \\
\leq & \ max(\|P_{ij} - Q_{ij}\|, \ \forall i, j),
\end{aligned}
\tag{4}
$$

since the B-spline basis functions are nonnegative and sum to one. Equation (4) provides a simple mechanism to bound the maximum distance between surface S and the ruled surface $\hat{R}$ since,

$$
\begin{aligned}
Dist(S(u,v), \hat{R}(r,s)) & = \max_{u,\, v} \ \min_{r,\, s} \ \|S(u,v) - \hat{R}(r,s)\| \\
& \leq \max_{u,\, v} \ \|S(u,v) - \hat{R}(u,v)\| \\
& \leq max(\|P_{ij} - Q_{ij}\|, \ \forall i, j).
\end{aligned}
\tag{5}
$$

An algorithm to approximate an arbitrary tensor product surface as a set of ruled surfaces is derived in Algorithm 1.

In algorithm 1, the function **maxDistance**, based on equation (5), returns an upper bound on the distance between the original surface S and its ruled surface approximation. In [Elb3], we discuss the quality of the bound, the generalization of the distance computation (equation (4)) for rational surfaces, and possible approaches for improvements on the distance bound in non uniform parametrization.

Algorithm 1 returns a set of ruled surfaces that approximates the original surface S to within the required tolerance τ. Figure 1 shows an example of four consecutive stages of Algorithm 1.

Algorithm 1

```
Input:
    S(u,v), surface to be approximated using ruled srfs in v.
    τ, tolerance of piecewise ruled approximation to be used.

Output:
    R, Set of ruled surfaces, approximating S(u,v) to within τ.

Algorithm:
    RuledSrfApproximation( S, τ )
    begin
        C1(t), C2(t) ⇐ Vmin and Vmax boundary of S;
        R ⇐ ruled surface between C1(t) and C2(t);
        R̂ ⇐ R refined and degree raised in v to match S;
        If ( maxDistance( S, R̂ ) < τ )
            return { R };
        else
        begin
            Subdivide S into two subsrfs S^1, S^2 along v;
            return
                RuledSrfApproximation( S^1, τ ) ∪
                RuledSrfApproximation( S^2, τ );
        end
    end
```

3 The Toolpath Generation Algorithm

A ruled surface can be side milled with high precision, by approximating an offset [Elb91] of tool radius to the ruled surface. The axis of the tool is then aligned with the ruled parametric direction v along $C_1(u) - C_2(u)$ (equation (3)), of the offset of the ruled surface. With the aid of Algorithm 1, one can approximate an arbitrary surface with a set of ruled surfaces. With the aid of equation (5), the error of the approximation can be globally bounded. The toolpath for each offset approximation of each ruled surface

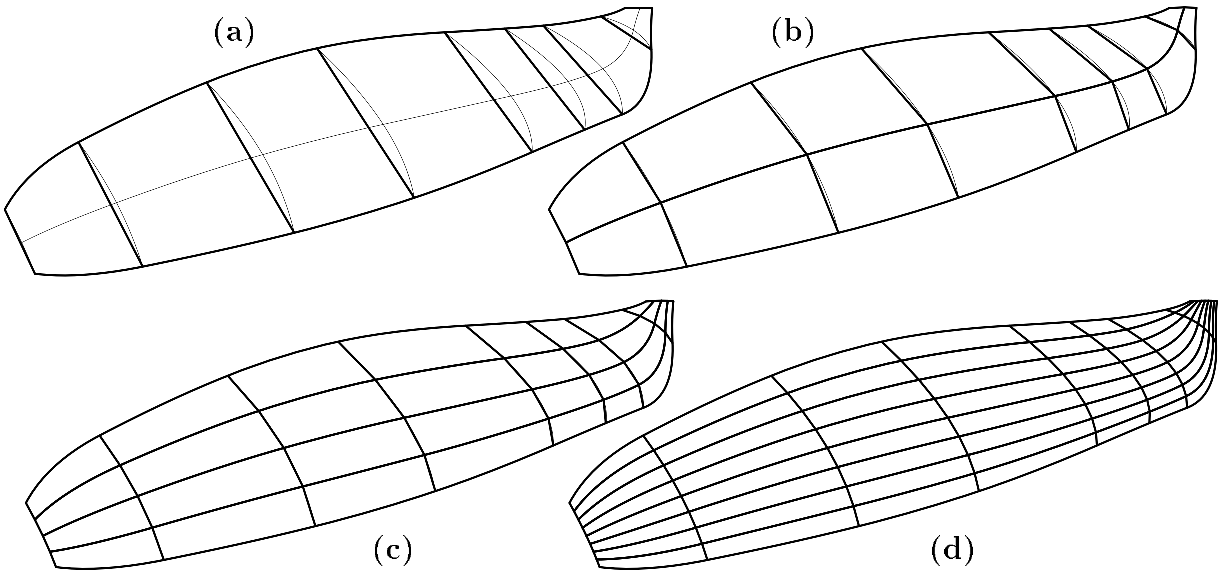

Figure 1: Four stages in approximating a surface with piecewise ruled surfaces (Alg. 1).

is then generated so that the side of the mill is tangent to the ruled surface, in the ruled parametric direction v.

Algorithm 1 can be used to generate a piecewise ruled surface approximation for a concave surface. Unfortunately, the side milling tool will gouge into the surface at every location. However, unlike 5-axis flat-end milling operation, the piecewise ruled surface approximation can be employed to generate locally gouge free toolpaths for machining hyperbolic or parabolic surface regions. Exploiting differential geometry, exactly one of the two principal curvatures is negative (zero), at an hyperbolic (parabolic) point [DoCa76]. For convex-elliptic, hyperbolic, and parabolic regions, there always exist a locally gouge free direction, in the tangent plane of the surface, that can be used to orient the axis of the tool (see Figure 2). That direction is a principal direction that is associated with a negative (zero for a parabolic point) principal curvature. The piecewise ruled surface approximation should be computed so that the ruled direction never has a positive normal curvature on the surface, to create a locally gouge free toolpath.

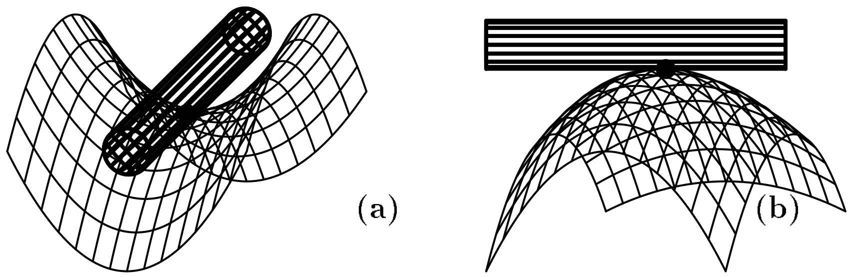

Figure 2: A saddle-like surface tool can be side milled (a) from a subset of directions in the tangent plane of the surface, without *locally* gouging into the surface, while a convex surface can be side milled (b) from any direction in the tangent plane of the surface without *locally* gouging into the surface.

An algorithm to generate a side milling toolpath can now be derived and is shown in Algorithm 2. The surface offset approximation stage and the piecewise ruled surface approximation stage (stages (1) and (2) in Algorithm 2), may be interchanged. One can consider computing the offset to the input surface S and then apply the piecewise ruled surface approximation to the offset surface. Alternatively, the piecewise ruled surface approximation can be applied directly to S and then offsets of ruled surfaces are computed as a second stage. In our implementation, discussed in section 4, the offset is first computed on S.

4 Examples

A simple example of a toolpath created using Algorithm 2 can be seen in Figure 3. Shown are the original surface, which is a part of a turbine blade, and the toolpath created from the offsets of the ruled surface approximation.

A computer model of a propeller of a model airplane was also used as a test case. The model is shown in Figure 4. Figure 5 shows the toolpath generated for the top

Algorithm 2

```
Input:
    S(u,v), surface to be machined.
    τ, tolerance of piecewise ruled approximation in v direction.
    r, tool radius for the side milling.

Output:
    𝒯, Set of toolpath curves to machine S(u,v) to within τ.

Algorithm:
    RuledToolPath( S, τ )
    begin
(1)     𝒮_r ⇐ S offset by r;
(2)     ℛ ⇐ RuledSrfApproximation( 𝒮_r, τ ) in v direction;
        𝒯 ⇐ toolpath derived directly from ℛ;
    end
```

section of the propeller's blade. The bottom section was machined in a similar way. This surface has both convex and saddle-like regions and the toolpath was generated so that the tool axis is roughly perpendicular to the major axis of the blade, preventing gouging. Plate 1 shows two machined propellers from aluminum with (top) and without (middle) their fixture along with an original wooden propeller (bottom).

A more complex example is shown in Figure 6. This blade of a marine propeller was similarly processed and approximated using a set of ruled surfaces. Figure 7 shows the piecewise ruled surface approximation for the top side of the blade while Figure 8 shows the derived toolpath. In Plate 2, the final part with the top side machined from aluminum is shown.

A relatively large tool can be usually used in this type of machining and surfaces. The low normal curvature in the direction orthogonal to the tool axis allows one to select

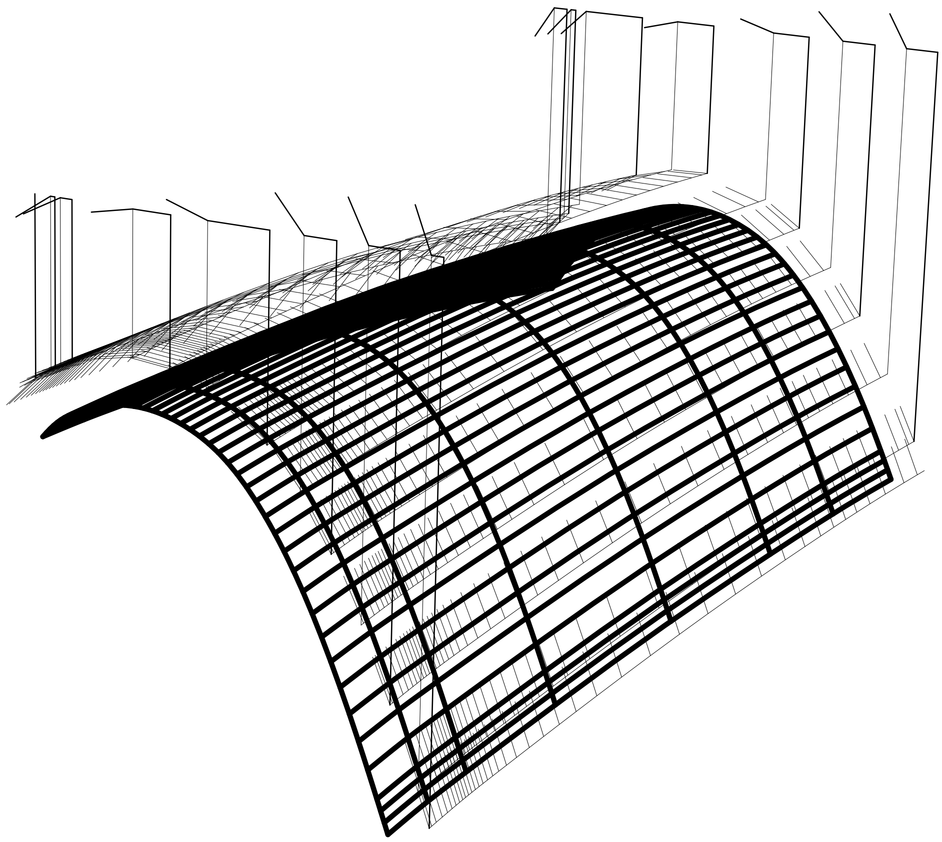

Figure 3: A simple example of the piecewise ruled surface side milling approach. The computed 5-axis side milling toolpath is shown with small vectors along the toolpath indicating the orientation of the tool. In this toolpath, the tool is retracted after each ruled surface path.

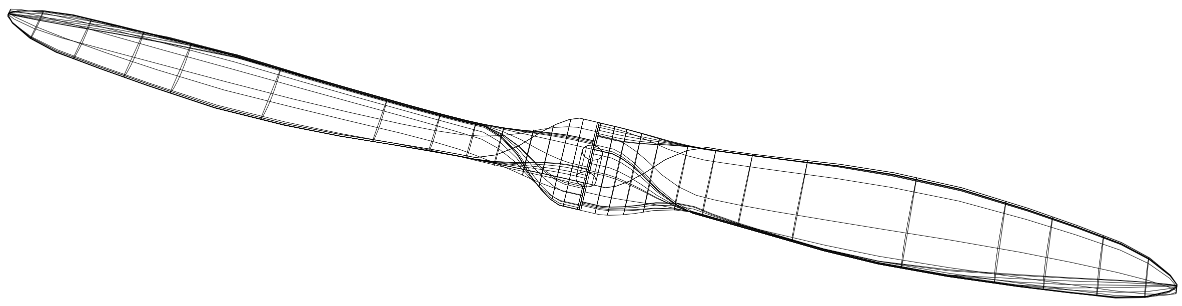

Figure 4: A propeller of a model airplane has blades with only convex and saddle-like regions which can be machined using the piecewise ruled surface side milling technique.

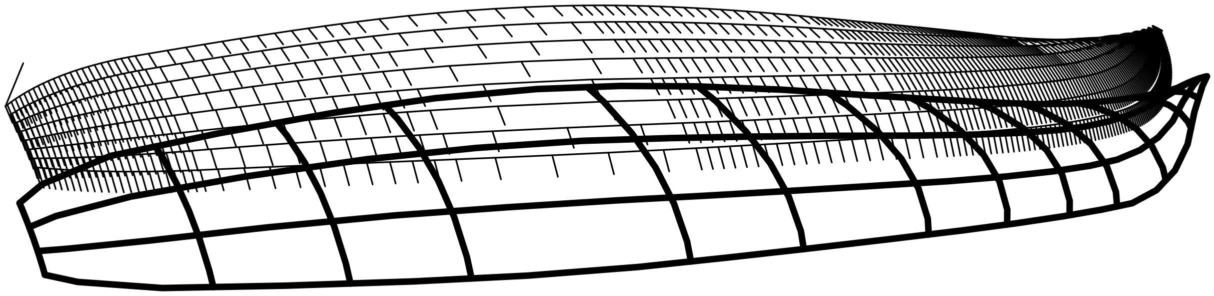

Figure 5: The toolpath for the top surface of the blade of the propeller in Figure 4. See Plate 1 for a picture of the machined propeller. Shown is a zigzag toolpath for which the tool is retracted after all ruled surface paths are being executed.

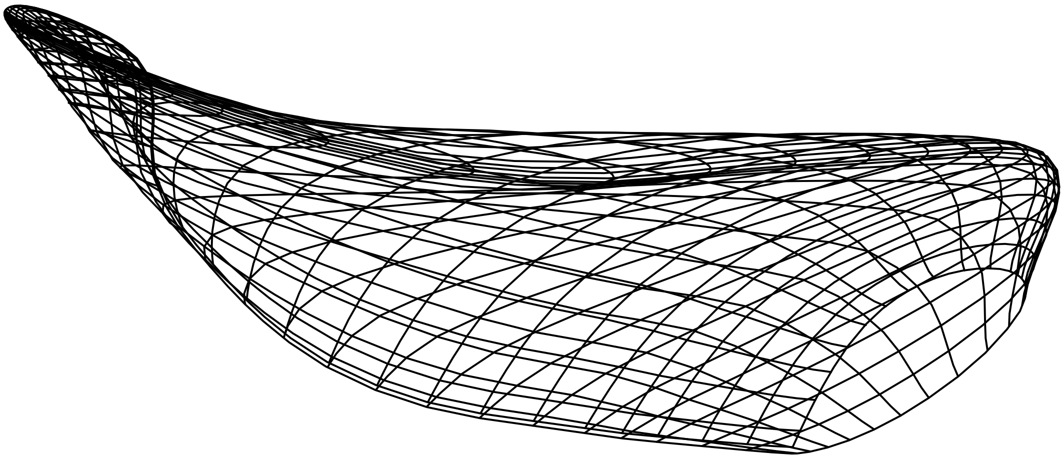

Figure 6: A blade of a marine propeller.

relatively large tools. For the propeller of the model airplane, a 0.75" (diameter) tool was used while for the blade of the marine propeller, a 1" tool was selected. As a result of selecting a large tool, the machining operations can be accelerated. Machining of the top part of the blade of the marine propeller took several minutes while manufacturing the entire propeller of the model airplane has been accomplished in far less than an hour, including manual (re)fixturing. In contrast, the toolpath generation time is negligible. A ruled surface approximation to a freeform surface can be constructed in a fraction of a second using high end workstations. Offset approximation of freeform surfaces is more

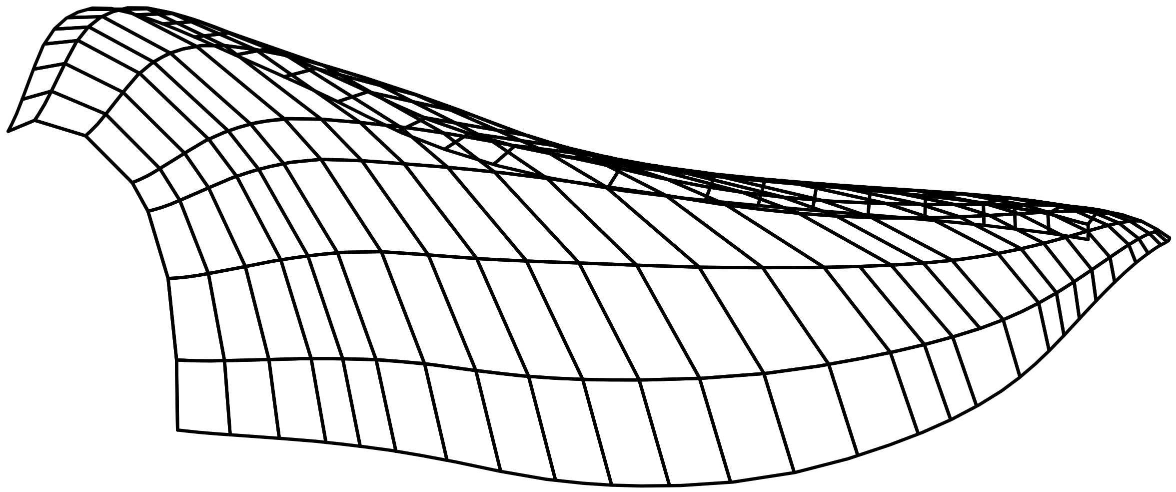

Figure 7: Piecewise ruled surface approximation of the top portion of the marine propeller from Figure 6. The approximation herein is about half as fine as is used to machine the aluminum part in Plate 2.

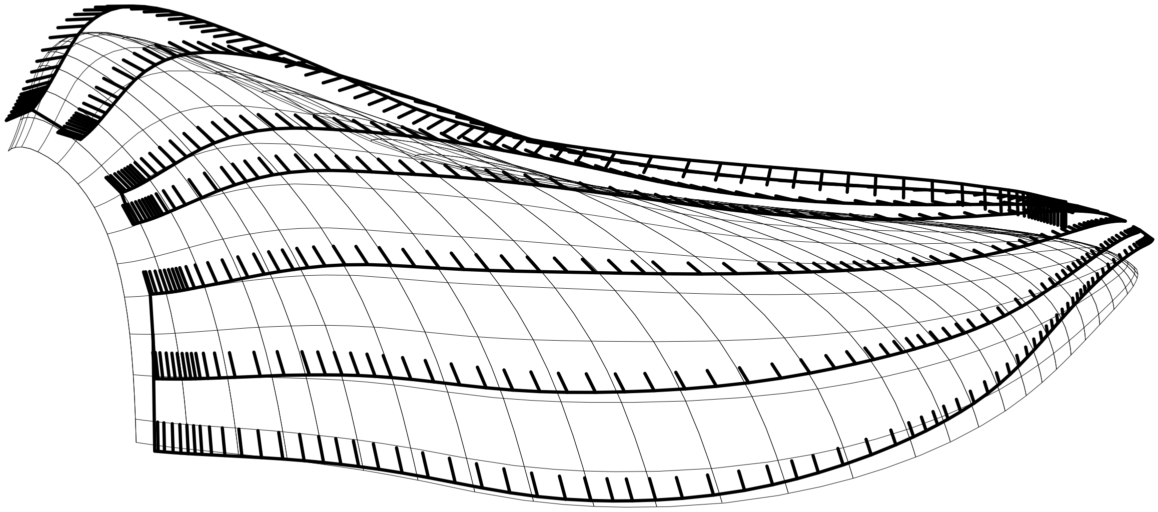

Figure 8: The toolpath for the top side of the blade of this marine propeller was generated using the piecewise ruled surface approximation technique (Figure 7). See Plate 2 for a picture of the machined part.

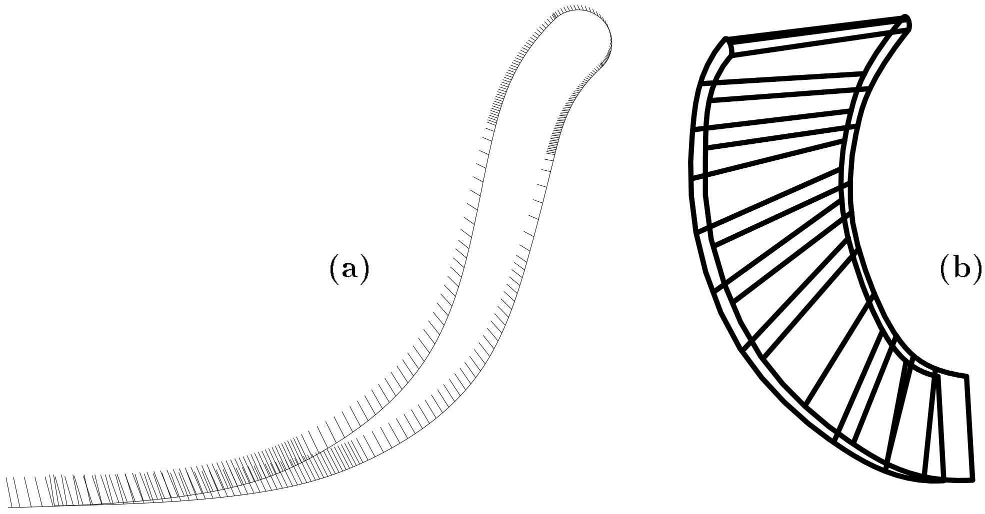

Figure 9: The toolpath for the side walls of the centrifugal compressor. The geometry (a) and the toolpath (b) are shown for different blades. In this case, the tool used was not cylindrical but tapered, shaped as a rounded-end truncated cone. See Plate 3 for a picture of the machined part.

time consuming and can be measured in seconds or even few minutes for highly complex surfaces. Yet, the toolpath generation time is far less than the needed machining time.

Algorithm 2, when applied to a surface that is ruled, generates a single pass toolpath as one would expect. The air flow tunnels for the centrifugal compressor in Plate 3 were machined using a tapered tool, shaped as a rounded-end truncated cone. The computation of the offset of the piecewise ruled surfaces must compensate for the angle of the tool axis and the line of tangency between the tool and the machined surface. Figure 9 shows the toolpath that was generated and the geometry of the wall of the tunnel, for two different blades.

5 Conclusion and Future Work.

This paper presents a method for machining saddle-like (hyperbolic) and convex surfaces using 5-axis side milling. This method extends the class of surfaces that can be machined in 5-axis mode from convex surfaces using flat-end tools, to convex and saddle-like surfaces using side milling.

In the examples of section 4, an iso-parametric direction for which the normal curvature is always negative was manually selected to align the tool axis. One can always evaluate the directions of the principal curvatures in the tangent plane of the surface and exploit them to compute the piecewise ruled surface approximation direction. The ruled direction can be set to be the direction in the tangent plane of the surface closest to a minimal negative principal curvature for the saddle-like case. Although this direction is not isoparametric, in general, the line of curvature of the minimal principal curvature can be computed arbitrarily precise using a piecewise linear approximation, or approximated by composing a curve approximating the line of curvature in the parametric space of the surface with the surface, Further, the maximal principal curvature can be employed to directly bound the largest tool radius that can be used for the machining operation of saddle-like regions, and hence to aid in the tool selection.

The toolpath generated using the piecewise ruled surface approximation consists of iso-parametric curves only. It is perceivable that the toolpath can be optimized using a similar technique to those used to generate the adaptive iso-parametric curve toolpath in [Elb93]. This direction should be further investigated.

Extending this algorithm to support trimmed surfaces is simple and can be accomplished by trimming the ruled surface approximation appropriately. Figure 10 shows a simple example of a trimmed saddle surface with its piecewise ruled surface approximation driven toolpath.

This paper does not address global accessibility issues for 5-axis machining operations. Several methods are known for the simulation, verification, and generation of 3-axis machining toolpaths for freeform surfaces. Most noticeably, Z-buffer and Z-depth

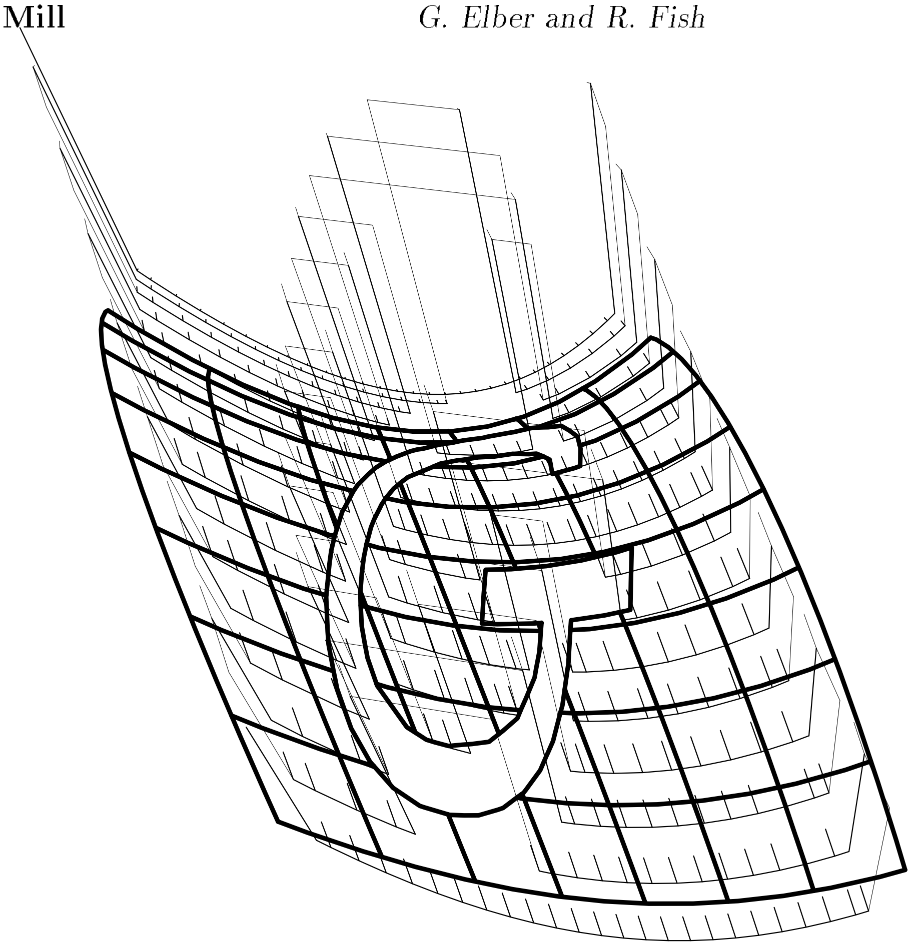

Figure 10: Piecewise ruled surface approximation of a trimmed saddle surface, from which a letter G was subtracted, is used to generate 5-axis side milling toolpath.

techniques [Sait91, Spen90] are becoming popular even in low end machines. Unfortunately, this problem is yet to be solved for 5-axis milling operations, so that 5-axis milling operations can be exploited to their full potential. Toward that end we define the following.

> **Definition 3** *A surface region, S, is considered* ***ball-end accessible*** *with respect to ball-end tool of radius r, if any point on S can be tangent to a ball-end tool of radius r so that the tool does not gouge into any other part of the model containing S.*

The computation of the accessibility of ball-end cutters is recognized as an important problem since it is used in traditional 3-axis and 5-axis ball-end machining. However, the side milling approach presented in this paper requires the derivation of a different accessibility consideration,

> **Definition 4** *A surface region, S, is considered* ***cylindrically accessible*** *with respect to a given cylinder of radius r, if any point on S can be tangent to the side of the cylinder so that the cylinder does not gouge into any other part of the model containing S.*

Herein, we only addressed *local accessibility issues*. The side of the tool might gouge into a ruled surface, if it is not developable, as we move away from the tool's contact point. The tool might also gouge into the original surface's geometry as we move away and the second order differential geometry analysis is not longer accurate. The computation of the accessibility of tools in 5-axis milling operations remains an intriguing and challenging problem. The solution to this problem will pave the way to the general use of 5-axis milling operations.

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Plate 1

Final machined examples of the model airplane's propeller. Shown are the original wooden propeller (top) along with two propellers machined from aluminum, without (middle) and with (bottom) their fixtures.

Plate 2

Final machined examples of the marine propeller's blade. Only top side of the blade was machined due to the difficulties in fixturing the blade.

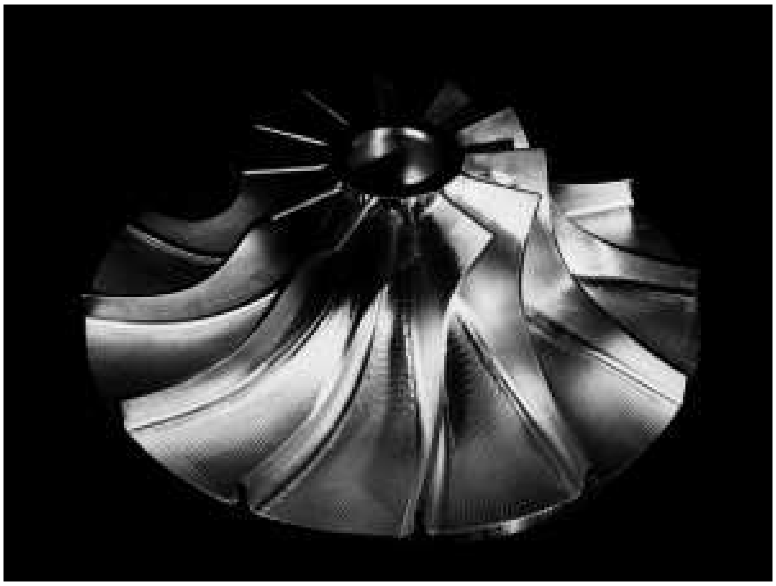

Plate 3

Final machined examples of the centrifugal compressor. The air flow tunnels in this part were machined using the piecewise ruled surface side milling technique and with a conical tool.