Efficient Nonlinear Optimization via Multiscale Gradient Filtering

Tobias Martin, Pushkar Joshi, Miklós Bergou, Nathan Carr

UUCS-12-001

School of Computing
University of Utah
Salt Lake City, UT 84112 USA

January 17, 2012

Abstract

We present a method for accelerating the convergence of gradient-based nonlinear optimization algorithms. We start with the theory of the Sobolev gradient, which we analyze from a signal processing viewpoint. By varying the order of the Laplacian used in defining the Sobolev gradient, we can effectively filter the gradient and retain only components at certain scales. We use this idea to adaptively change the scale of features being optimized in order to arrive at a solution that is optimal across multiple scales. This is in contrast to traditional descent-based methods, for which the rate of convergence often stalls early once the high frequency components have been optimized. Our method is conceptually similar to multigrid in that it can be used to smooth errors at multiple scales in a problem, but we do not require a hierarchy of representations during the optimization process. We demonstrate how to integrate our method into multiple nonlinear optimization algorithms, and we show a variety of optimization results in variational shape modeling, parameterization, and physical simulation.