Selected Fixed Point Problems and Algorithms

Ch. Booniasirivat, K. Sikorski,[a] Ch. Xiong

UUCS-07-007

[a]Partially supported by DOE under the C-SAFE center

School of Computing
University of Utah
Salt Lake City, UT 84112 USA

March 6, 2007

Abstract

We present a new version of the almost optimal Circumscribed Ellipsoid Algorithm (CEA) for approximating fixed points of nonexpanding Lipschitz functions. We utilize the absolute and residual error criteria with respect to the second norm. The numerical results confirm that the CEA algorithm is much more efficient than the simple iteration algorithm whenever the Lipschitz constant is close to 1. We extend the applicability of the CEA algorithm to larger classes of functions that may be globally expanding, however are nonexpanding/contracting in the direction of fixed points. We also develop an efficient hyper-bisection/secant hybrid method for combustion chemistry fixed point problems.

Keywords: fixed point problems, optimal algorithms, nonlinear equations, ellipsoid algorithm, computational complexity.

1 Introduction

We want to approximate a fixed point $\mathbf{x}^*$ that satisfies the equation $\mathbf{x}^* = f(\mathbf{x}^*)$ for a given function f. Many problems can be formulated as fixed-point problems. For example, a root-finding problem for nonlinear equations $f(\mathbf{x}^*) = 0$ can be rearranged as