A Hole-Filling Algorithm for Triangular Meshes

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Abstract

Data obtained by scanning 3D models typically contains missing pieces and holes. These can be caused due to scanning artifacts or artifacts in the surface due to wear and tear. We provide a method based on the Moving Least Squares projection to fill holes in triangular meshes obtained during the process of surface reconstruction . Our method can be applied to holes with non-planar geometry as well as small, planar holes.

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Data obtained by scanning 3D models typically contains missing pieces and holes. These can be caused due to scanning artifacts or artifacts in the surface due to wear and tear. We provide a method based on the Moving Least Squares projection to fill holes in triangular meshes obtained during the process of surface reconstruction . Our method can be applied to holes with non-planar geometry as well as small, planar holes.

1 INTRODUCTION

Scanning 3D data and reverse engineering data to obtain computer models has numerous applications in the field of CAD modeling and entertainment. However, during the process of scanning, several factors such as occlusion, low reflectance or even missing pieces in the original geometry can lead to incomplete data.

Holes in the data are not usually esthetically appealing while rendering. Further, situations often arise in computer graphics and modelling where only closed and watertight objects can be dealt with. Also, lack of data can cause problems during some surface fitting algorithms and surface parameterization techniques. Hence the problem of filling holes in scanned data is important.

2 PREVIOUS WORK

Most of the previous work on hole-filling is based on constructing an implicit surface from the data that interpolates the hole. A significant part of such research, integrates the process of hole filling into surface reconstruction. In [4], a surface reconstruction technique based on level sets is presented, that uses a variational approach and evolves the surface until it approximate the data adequately. This method inherently addresses the problem of holes. Also, most of the methods that are based on finding an implicit representation are based on finding a signed distance function. In [7], a volumetric method is presented to obtain cumulative weighted signed distance function from multiple aligned range images, in which the authors address the problem of hole-filling by space-carving to obtain watertight models. This hole-filling method however produces unwanted geometry in certain cases and aliasing artifacts which are eliminated by post filtering the reconstructed mesh using weighted averaging.

In [5] poly-harmonic radial basis functions are used to fill holes by fitting an RBF to the signed distance function and extracting the iso-surface.

Treating hole filling as a post-processing step has the advantages that the hole filling process can be used independently with any reconstruction method and can handle holes that arise from situations other than scanning defects. Also such a method can focus on operating near the vicinity of the hole and reduce computation time. Some work has been done on extending image inpainting [3] to surfaces that treats hole-filling as a post processing step. In [8] a method using volumetric diffusion is presented that obtains a volumetric representation of the data, a 3D grid of values of signed distance function, and fills holes using a diffusion process of alternate blurring and compositing that operates only in the vicinity of holes.

In [10] another slightly slower method is presented, based on image inpainting, that uses a system of coupled anisotropic partial differential equations.

In [1], the MLS projection procedure is used for adequately sampling of points in a point cloud for rendering purpose. However, the authors focus on efficiently dealing with undersampling and oversampling in the input and not on hole-filling. In [11], a mesh based hole-filling algorithm is presented. In this method, a best fit plane in the vicinity of the hole is found and data is parameterized by projecting the points orthographically onto this plane. Then data is sampled in the parametric domain and the moving least squares technique is used to interpolate data at these points. This approach is efficient to implement. However it can only handle holes of simple geometry that resemble a plane since it relies on parameterizing the vicinity of the hole by orthographic projection onto a plane. While our method is similar to this method, it can handle holes with non-planar geometry through its local technique.

2.1 MLS PROJECTION PROCEDURE

The MLS projection procedure was proposed by [9] and [1] to deal with meshless surfaces. Given a point set, the MLS projection operator projects a point r near the surface onto the surface implicitly defined by the set of points. This surface can be defined as the set of points that project onto themselves.

The MLS projection operator proceeds in two steps. To project a point r, the first step requires finding an optimal local reference plane for the neighborhood of r by minimizing the perpendicular distance of each point p_i in the neighborhood. If n is the normal to the plane and t the distance of the plane from r, $\sum_{i=1}^{N} \langle n, p_i - r - tn \rangle^2 \theta(||p_i - r - tn||)$ is minimized with respect to n and t. This is a non-linear minimization process (figure 1). A local parameterization is obtained by projecting each point in the neighborhood onto this reference plane. The next step involves fitting a bi-quadratic polynomial g, $\sum_{i=1}^{N} (g(x_i, y_i) - f_i)^2 \theta(||p_i - q||)$ where $q = r + tn$ (q is the projection on the best fit plane), (x_i, y_i) are the parameter values of p_i in the local reference plane and f is orthogonal to the local reference plane, to approximate the surface locally using the moving least squares technique. This polynomial when evaluated at the point q, gives the desired MLS projection.

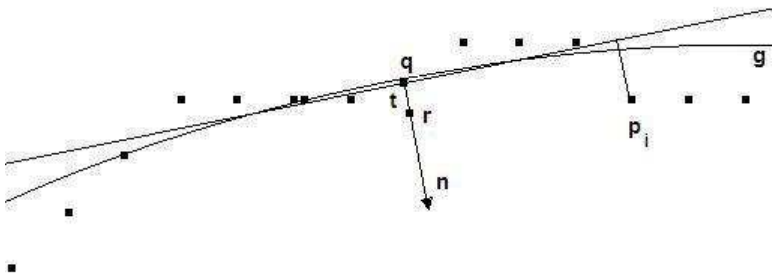


Figure 1: MLS Projection procedure

3 OUR HOLE FILLING ALGORITHM

We present a new hole-filling procedure to fill holes in triangular meshes based on the MLS projection procedure. Our method operates only in the vicinity of the holes. Hence it is efficient in terms of computation time. Our method can be used with any surface reconstruction algorithm to fill holes as an independent post-process and is easy to implement.

Since we operate on triangular meshes for the basic data type, it is straightforward to find the boundaries of the hole by finding a connected set of edges that are associated with only a single triangle. This algorithm can be extended to deal with holes in point clouds.

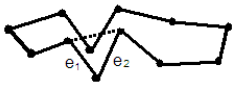
Since a global parameterization of the data is not available, the filling procedure needs to be local in nature to handle holes of arbitrary geometry. The algorithm consists of two alternating steps, the *make-convex* step and the *add-vertices* step and proceeds until the boundary size reduces to three vertices and the hole can be filled by inserting a single triangle. The *make-convex* step involves making a new triangle by adding an edge between adjacent pairs of edges in the boundary that make an angle lesser than ϕ. The *add-vertices* step involves adding new vertices along the boundary of the hole closing the hole radially inwards at each edge. This section discusses these two steps in more detail.

***make-convex*:**
For every pair of adjacent edges b_1 and b_2 on the boundary, such that the angle between the edges is less than ϕ

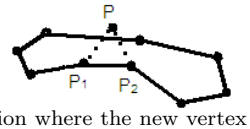
1. A local neighbourhood of the mid-point of the edge is obtained. A local reference plane is computed with respect to the mid point of the edge (as presented in [9] and described in section 2.1) .

2. A local parameterization is obtained by orthographically projecting the local neigbourhood onto the plane.

3. If a new edge joining b_1 and b_2 in the parametric domain does not intersect any other boundary edge in the parametric domain that lies within the neighbourhood, the new edge is added to the mesh. A situation where this condition is violated is shown in figure 2(a).



(a) A situation where the new edge introduced in the *make-convex* step crosses other neighbouring boundary edges in the parametric domain



(b) A situation where the new vertex introduced in the *add-vertices* step crosses other neighbouring boundary edges in the parametric domain

Figure 2: A figure showing the situations that can arise if the various checks are not performed

Since new boundary edges are introduced in this process, multiple passes of this step are made until no adjacent boundary edges make an angle greater than ϕ. The results of an intermediate stage of application of this process is shown in 3.

***add-vertices*:**

1. For every edge e in the current boundary

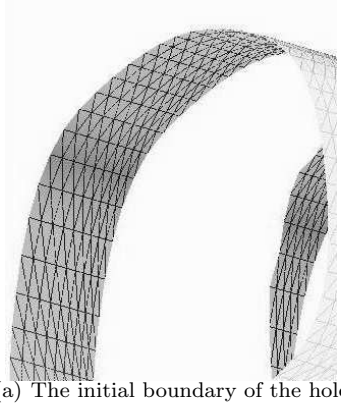
   (a) A local neighbourhood of the mid point of the edge is obtained. A local reference plane is computed as outlined in section 2.1 with respect to the mid point of the edge e.

   (b) A local parameterization is obtained by orthographically projecting the local neigbourhood onto the plane.

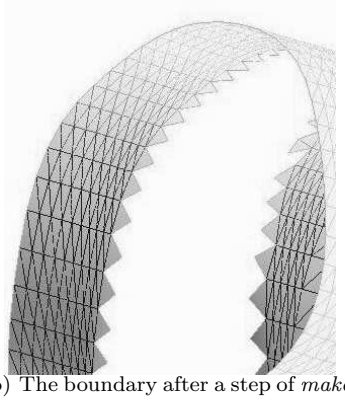
   (c) A new point along the perpendicular bisctor of e in the local parameterization is chosen, at a specified distance d_e.

   (d) A new point on the surface is computed using a local MLS approximation.

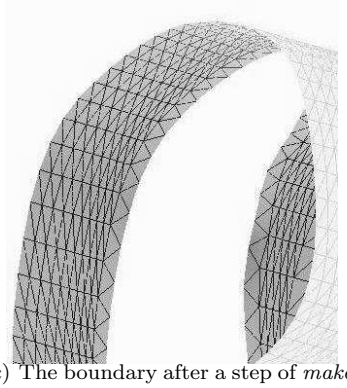
2. For every new point p, the closest edge in the current boundary is found. Let the end points of the edge be p_1 and p_2. In order to ensure that well behaved triangles



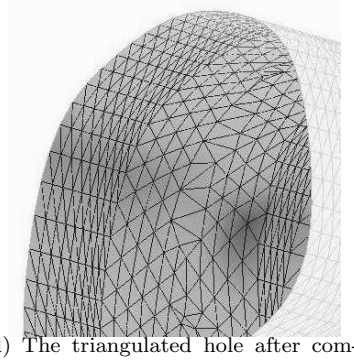
(a) The initial boundary of the hole



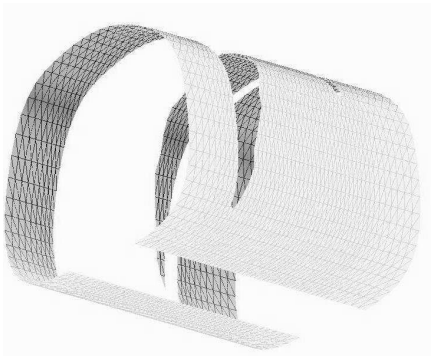
(b) The boundary after a step of *make-convex* and *add-points*



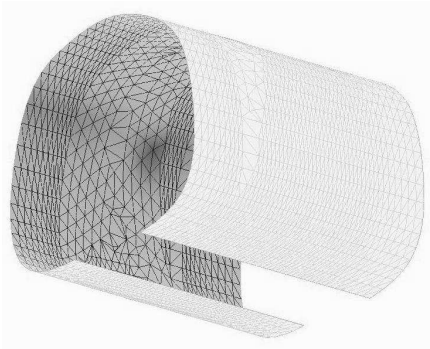
(c) The boundary after a step of *make-convex*



(d) The triangulated hole after completion of the hole-filling process



(e) A complete view of the surface with the hole



(f) A complete view of the filled surface

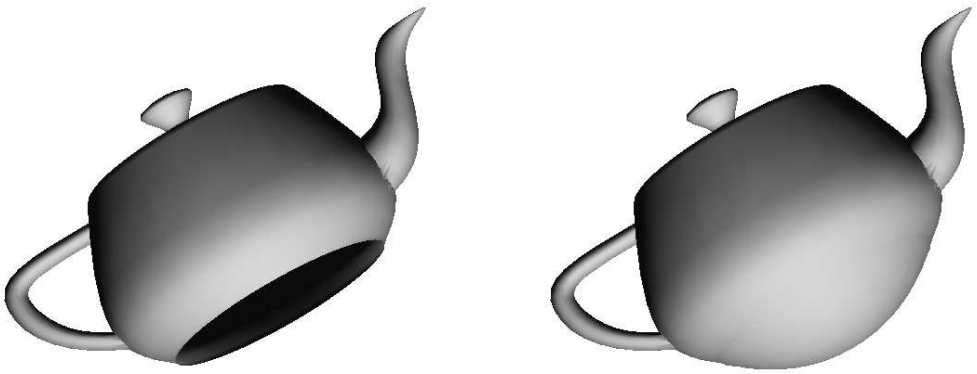
Figure 3: A figure showing the various steps in the hole filling process.

are obtained, a check is made to see if introducing a new triangle t with p, p_1 and p_2 crosses any other boundary edge in the local parametric domain. If so, the point p is discarded. Such a case is shown in figure 2(b). If not, the triangle t is introduced and the boundary updated. This process is shown in figure 3(c).

If no new vertices or edges have been introduced in the boundary after an iteration of *make-convex* followed by *add-vertices*, and the boundary size is greater than 3 verties, the process is repeated with a reduced d_e and ϕ to promote progress.

The choice of d_e and the value of ϕ play a significant role in determining the shape of the triangulation. If ϕ is too large, ill-shaped triangles result, that are elongated in one direction. We have obtained reasonable results for ϕ of about $5\pi/9$. Some alternatives for choosing d_e are to select a d_e so that the resulting triangle is equilateral in the parametric domain with side e, where e is the side under consideration, or to create an isoceles triangle in the parametric domain with two of the sides as the average edge length a. We choose the later method and choose d_e as $\sqrt{(4a^2 - e)}/2$.

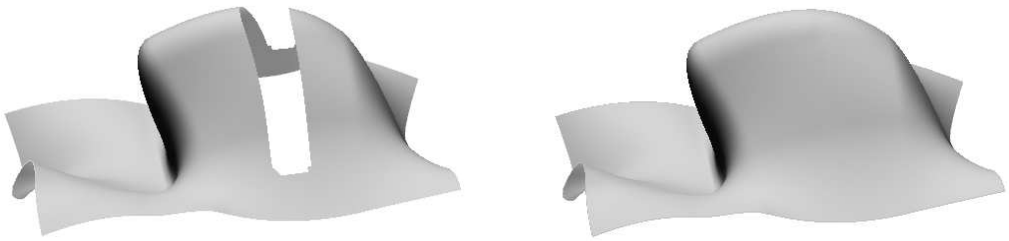
4 RESULTS



(a) Teapot model with a hole at the bottom

(b) Teapot model with the hole smoothly filled

Figure 4:

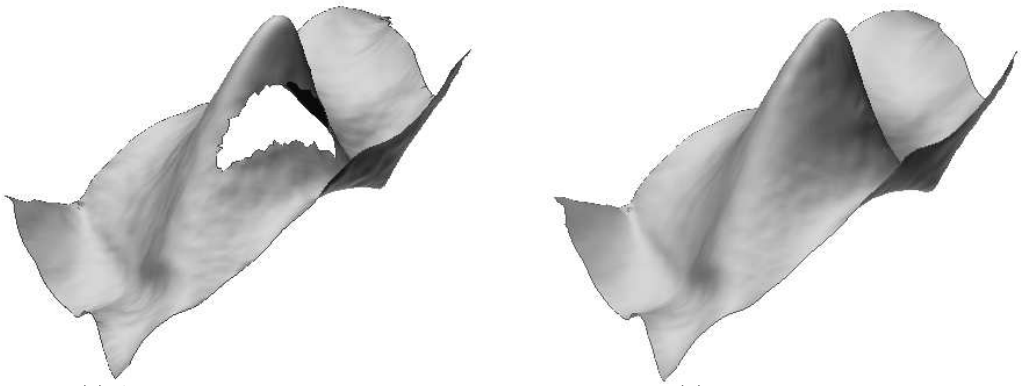


(a) A surface with a hole whose vicinity cannot be parameterized by projection

(b) The non-planar hole filled using our method

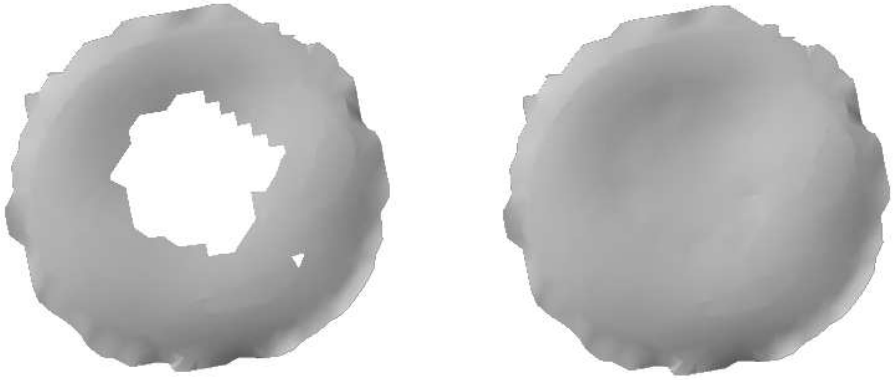
Figure 5:

This section presents some more examples of our hole filling process. Figure 4(a) shows the Utah teapot with a hole at the bottom. In figure 4(b) this hole is smoothly filled using our method. In figure 5(a) a non plannar hole is shown which cannot be parameterized by



(a) Another surface with a hole (b) The filled surface

Figure 6:



(a) A hole in a wheel scanned from a tank model (b) The hole filled

Figure 7:

projection. Also mesh based methods that attempt to find the best fit plane in the vicinity of the hole using a PCA analysis tend to produce a local plane that lies along the direction of the hole. But parameterizing the vicinity of the hole by projecting onto this plane does not result in a single valued mapping. In figure 5(b) this hole is filled using our method. Because our method fits a smooth surface in the vicinity of the boundary and interpolates to find new points it ensures that there are no visual discontinuities at the boundary. Figures 6(a) and 7(a) show two more surfaces with holes and 6(b) and 7(b) show the surfaces filled with our algorithm.

5 CONCLUSION

We have presented a new mesh based hole-filling algorithm that can be used to fill holes in triangular meshes based on the MLS projection procedure. It can handle non-planar holes as it operates based on local parameterizations rather than a global one. Finally our method is simple and efficient since it operates only in the vicinity of holes.

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