Reaction-Diffusion Patterns in Smart Sensor Networks

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Abstract

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Abstract

We introduced the use of Turing's reaction-diffusion pattern formation to support high-level tasks in smart sensor networks (S-Nets). This has led us to explore various biologically motivated mechanisms. In this paper we address some issues that arise in trying to get reliable, efficient patterns in irregular grids with error in inter-node distances.

1. Introduction

Sensor networks are a crucial part of the IT infrastructure, and intelligent utilization of them is of increasing importance. Sensor network research to date has focused mainly on the development of new sensing technologies and the systems aspects (OS, network communication, security, etc.) of such networks. Much remains to be done at the higher level of information extraction, interpretation and exploitation of networked sensor systems. Our central thesis is that bio-based engineering will lead to strong solutions in this domain; that is, we propose to identify and ultimately incorporate effective computational strategies used by biological systems. The challenge is to identify mechanisms that lead to algorithms or paradigms that are reliable, inexpensive and ubiquitous in many applications.

Others have explored the use of both reaction diffusion and more general diffusion methods in computer vision and robotics. For example, Fukuda et al. describe the use of reaction-diffusion techniques in robot motion[7]. Moreover, as described by Peronna et al.[30], multi-scale descriptions of images (i.e., scale-space) can be done by embedding the original image in a family of images obtained by convolving the original image with a filter; Koenderink[18] have shown that this is equivalent to finding the solution of the diffusion equation:

$$I_t = \bigtriangledown^2 I = I_{xx} + I_{yy}$$

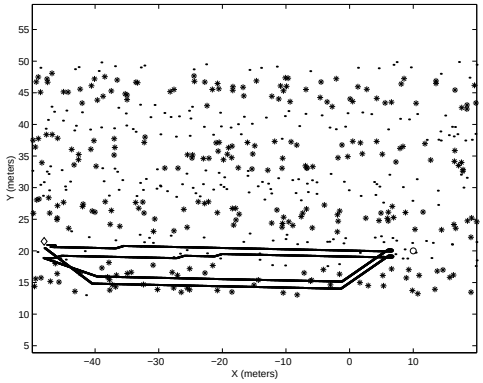

Figure 1. Robot Path in Reaction Diffusion Pattern (○ is the fire control point; ◇ is the robot load point)

We believe that it will be quite useful for S-nets to use similar methods to analyze sensed data of various sorts.

For example, consider a forest fire scenario: sensor devices are dropped into a wide geographic area, establish a network, compute coordinate frames, calculate gradients, and produce a stripe pattern of off-on signals that can be used by fire fighting agents to go to a fire control point by following *on* devices (pattern == 1) and return by following *off* devices (pattern == 0) (see Figure 1). Such patterns can be computed by very robust reaction-diffusion systems derived from models of biological pattern formation.

Our general research program is to explore a small set of biological mechanisms, and we hope to make significant contributions by providing (1) biologically realistic models and efficient computational counterparts, (2) fault tolerant frameworks in which to run them, and (3) demonstrations of their application in human interface and large-scale sensor networks. In addition, we are building Smart Sensor Network simulation, emulation, and experimentation testbeds [13]. Here we describe some initial results in the first of

these areas.

2. Background

Sensor networks have received increasing attention over the last few years. For example, DARPA's SensIT program envisioned long-lived, cheap sensor devices [19]. David Culler's work on sensor networks explores the rich design space of low-power processors, communication devices and sensors [14]. NSF has recently funded an STC Center for Embedded Network Systems headed by Deborah Estrin that will develop algorithms for wireless and distributed sensing systems [26].

Some examples of issues addressed by these various projects include: power minimization [32, 35], self-configuration [4, 21], data handling [3, 15, 22], systems issues [8, 31, 36], and fault tolerance [36]. In general, higher-level exploitation of sensor networks applies standard sequential or distributed algorithms to the data. Some work in this area includes calibration [34] and habitat monitoring [24].

Our own work started in the late 90's [12], and has mainly addressed the creation of an information layer on top of the sensor nodes. This includes distributed algorithms for leadership protocols, coordinate frame and gradient calculation, reaction-diffusion pattern formation, and level set methods to compute shortest paths through the net [5, 6, 11].

The information processing issues include the representation of information and knowledge, the processing of that information, and the development of efficient, robust, scalable algorithms. Our thesis is that the exploitation of distributed sensor and communication devices by a team of mobile robots or agents offers performance and capability advantages in terms of speed, energy, robustness and communication requirements. We are developing capabilities to perform cooperative computations and provide local and global information about the environment.

3. Reaction Diffusion Patterns in S-Nets

The biological thrust of our work is to model and understand sensing and signaling in biological systems, and to ascertain how that can be applied to smart sensor systems. Specifically, we incorporate known bio-based information processing strategies and network architectures in order to improve S-Net organization and information processing capabilities.

We have previously described pattern forming reaction-diffusion methods for S-Nets [5, 6]. Other researchers have proposed diffusion models as well; for example, [16] proposes directed diffusion - a datacentric communication coordination technique that "enables energy savings by selecting empirically good paths and by caching and processing data in- network." The focus of such work is more on the networking and operating systems aspects of the sensor network, whereas our work is more concerned with the sensor network as a computation engine itself. More closely related to our work is that of Justh and Krishnaprasad [17] who propose the active coordination of a large array of microactuators by means of diffusive coupling implemented as interconnection templates, and Nagpal [28] who describes methods to create patterns of diverse geometry. We believe that this style of research will reap great benefits in 2 aspects: (1) network *morphogenesis*, and (2) sensed data analysis.

As Meinhardt points out [25], "the control of development in a higher organism is one of the major unresolved problems in biology ... in a developmental system a signaling and signal-receiving mechanism must exist which enables the cell to communicate in a manner appropriate to its position ... [the] goal is to show which interactions of substances can lead to such signaling systems and how the cells then can respond to these signals in order that stable states of determination are attained." This matches our view of the core issues, and we see that their solution can heavily impact sensor network algorithms as well. For a recent collection of work on reaction-diffusion pattern formation, see Maini and Othmer [23].

To date, we have assumed a dense set of sensors and have found it useful to develop a pattern in the network by means of a distributed reaction-diffusion mechanism. For example, striped patterns can be formed along the temperature gradient so that mobile robots can move along the white stripe toward a fire and along a black stripe to return to base (see Figure 1). We use Turing [33] and other reaction-diffusion mechanisms [27] to generate such patterns in S-Nets. The basis of this mechanism is a set of equations that captures the reaction and diffusion aspects of certain chemical kinetics:

$$\frac{\partial \mathbf{c}}{\partial t} = f(\mathbf{c}) + D\nabla^2 \mathbf{c} \tag{1}$$

where $f(\mathbf{c})$ describes the reaction and $D\nabla^2 c$ expresses the diffusion component. The simplest such systems have two *morphogens* or variables; one of these acts as the activator and the other acts as the inhibitor (i.e., some cells will have more morphogen over time, and other cells less – this uneven distribution leads to the pattern). The two variable system can be modeled by:

$$\frac{\partial u}{\partial t} = \gamma f(u,v) + \nabla^2 u, \frac{\partial v}{\partial t} = \gamma g(u,v) + d\nabla^2 v \tag{2}$$

where u and v are the concentrations of the morphogens, d is the diffusion coefficient and γ is a constant measure of scale. The functions $f(u,v)$ and $g(u,v)$ represent the reaction kinetics. As an example, we have explored the generation of spatial patterns using the Thomas system of

equations [27]:

$$f(u,v) = a - u - h(u,v), g(u,v) = \alpha(b - v) - h(u,v)$$

$$h(u,v) = \frac{\rho u v}{1 + u + K u^2} \quad (3)$$

where a, b, α, ρ, and K are the positive reaction parameters. They define a domain in which the Thomas equations become linearly unstable to certain spatial disturbances (noise). This domain is referred to as Turing space where the concentrations of the two morphogens will become unstable and result in the stripe patterns. The pattern is the result of each network device running the equations locally while diffusing to its neighbors; a stable solution is thresholded to produce a binary value at each sensor, and the total of these gives the pattern in the S-Net.

Patterns in the S-Net can be used to support many high-level algorithms or activities:

- stripe, spot or ring patterns can be used as encoders for physical or logical purposes; for example, a robot can keep track of how far it has traveled (physical), or communication packets can travel along certain stripes to minimize power cost or to avoid congestion (logical)
- certain sets of patterns form a basis set for 2D images (e.g., Haar or Hadamard basis sets); any map (topo, etc.) or image can then be encoded in terms of the coefficients associated with the respective basis images.
- the patterns can be used as a reference wave so that sensed data (or features derived from it) can be encoded as an interference pattern (i.e., a hologram)
- moving waves can also be computed, and thus the S-Net can serve as a signal carrier or modulator.

Understanding the precision and reliability of pattern formation is then of high importance.

4. Relevant Issues in Pattern Formation

Some work has already been done to determine the range and type of patterns possible with the Turing pattern formation approach. Theoretical aspects have been studied and regions of the parameter space characterized as they relate to pattern formation (i.e., the parameters are the coefficients in the PDEs) [1, 10, 20]. Others have investigated how pattern formation is influenced by number of cells, time scale, and initial condition variation. In particular, Bard and Lauder [2] showed that "stable repeating peaks of chemical concentration of periodicity 2-20 cells can be obtained in embryos in periods of time less than an hour. We do find however that these patterns are not reliable. Small variations in initial conditions give small but significant changes in the number and positions of observed peaks." They showed that this method has difficulty producing exact patterns reliably. We have found other difficulties in producing the patterns necessary to support higher-level tasks. We describe these here and propose some solutions.

4.1 Topology vs. Metrology

A major issue is that most of the reaction-diffusion work is based on the *topological* configuration of the cells; that is, diffusion occurs at the cell level and patterns are formed at the cell level. Thus, the equations are solved for cells - and not at specific locations in space. This means that even though stripes exist, their width is measured in cells - not in units of length (e.g., meters). For the applications that we envisage, e.g., using stripes as distance encoders for mobile agents who use the S-Net, it would be more useful for the stripe dimensions to be related to physical units of length.

One possible solution to this problem is to exploit the physical dimension of the cell (in our case an S-Net device). The dimension of the stripe can be tied statistically to the number of nodes per unit area. Assume that there are n^2 devices per unit area; then there are n devices per unit length (assuming a uniform distribution). Thus, if there are s devices per stripe width, then the width of a stripe is given as:

$$w = \frac{s}{n}$$

where w is the width in physical units. We have found this to be a useful approximation.

4.2 Non-Uniform Placement of Cells

Another major factor in the nature of the pattern, and even its possibility of forming, is noise and error in the various parameters of the system. As Bard and Lauder [2] have shown, small variations in initial conditions yield significant changes in the patterns formed. However, since we get to initialize the morphogen concentrations in our computational context, this problem can be countered by using specific initial values to achieve the required patterns.

A more significant issue for us is that the reaction-diffusion pattern formation equations assume that the inter-cell distance is uniform (and usually equal to 1). Our S-Nets, however, do not form a uniformly spaced grid in 1D or 2D; in fact, we generally assume that the sensor devices are randomly dropped in the environment. In addition, the diffusion part of the equations uses the inter-node distances in the computation of the second derivative. Two concerns are:

- these distances are not uniform, and

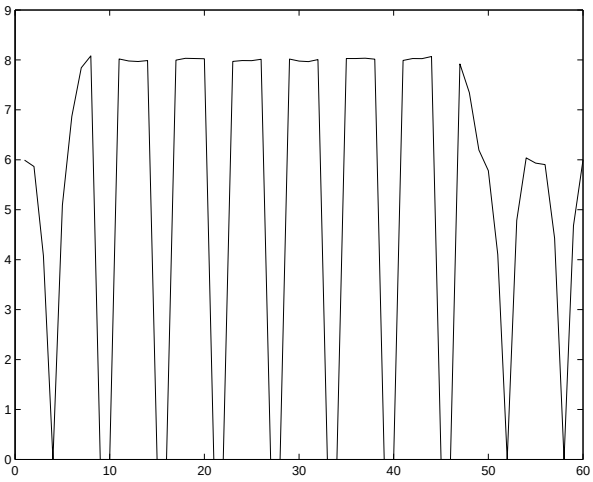

Figure 2. Basic Turing Pattern in 1D

	10 cells	20 cells	60 cells
a mean value	1.84	9.31	19.22
a stdev	1.2849	1.8185	1.3377
b mean value	1.14	9.49	19.27
b stdev	1.2872	1.0683	1.1534

Table 1. Mean and Stdev for Morphogens a and b

- in an actual implementation, there will be some amount of error in the inter-node distance determination.

This has led us to investigate the impact of non-uniform spacing on the pattern computation.

The basic 1D Turing reaction-diffusion mechanism produces a pattern as shown in Figure 2, and takes about 700 iterations to converge. A set of 1,000 experiments were run with different initial conditions for 10, 20 and 60 cells in 1D. Table 1 gives the results for the mean and standard deviation of morphogens a and b.

We also investigated the frequency of the stripe pattern on equispaced points; Figure 3 gives the mean and standard deviation of the pattern frequency on 20, 60 and 120 points over 1,000 trials. A reaction is considered to have converged if morphogen b changes by less than 10^{-4} on 5 consecutive iterations.

Next consider what happens when error is introduced into the inter-device distances (this is the same as simply having irregularly spaced points since the distance, although in error, will be calculated once at the start of the S-Net formation and stay the same thereafter). The point locations are determined as follows:

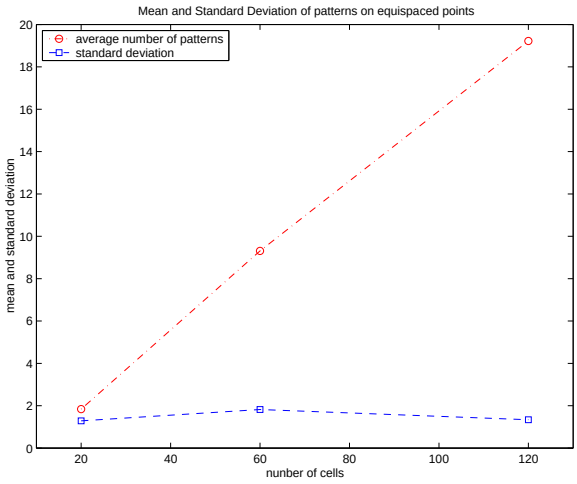

Figure 3. Mean and Standard Deviation of Frequencies of Patterns

random error	total iterations	frequency
0	830	10
0.05	450	10
0.1	320	10
0.2	190	8
0.3	130	8
0.4	130	6

Table 2. Position Error vs. Time to Converge and Stripe Frequency

- start with 60 equispaced points, 1 unit length apart,
- add uniform noise to the location as: $\pm p\%$ for $p = 5, 10, 20, 30, 40$.

Table 2 gives the total number of iterations for the pattern to emerge visually and the stripe frequency for a given amount of error in the inter-device distances. Beyond this number of iterations, the amount of morphogen in some cells grows without bound. Note that as error increases, the stripe frequency decreases.

Figure 4 shows an interesting result: the number of iterations required to converge decreases with an increase in error. Thus, it is more efficient to have non-uniformly spaced points, and the larger the variation in the inter-node distance, the faster the convergence. We have investigated the use of Chebyshev points (60 of them), and the reaction process converged in 3000 iterations. The frequency is somewhat higher using Chebyshev points, though (see Figure 5). points. Figure 6 shows a typical pattern formed with 30 % error in the inter-node distance. However, the fact that

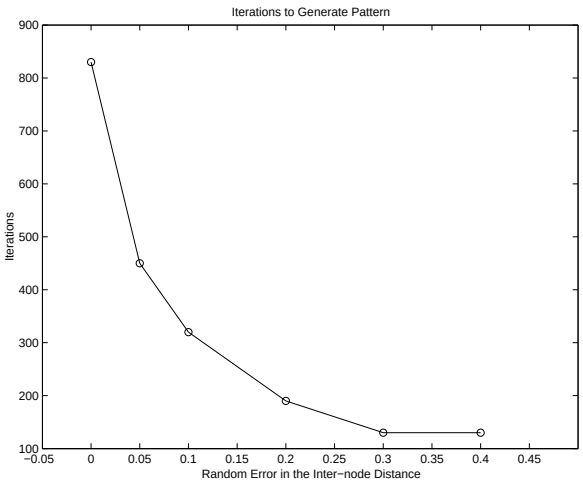

Figure 4. Number of Iterations required to generate patterns with random errors in distances.

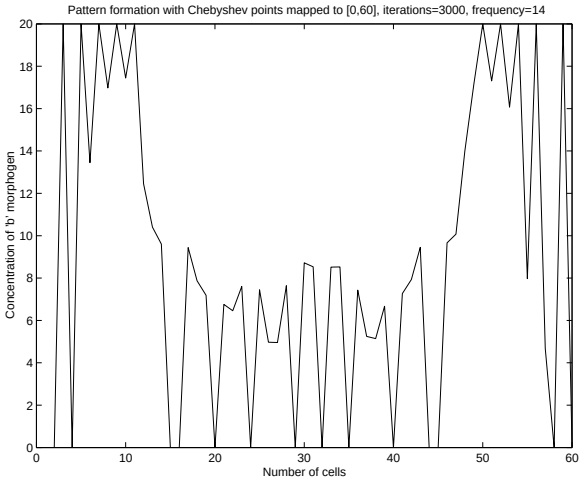

Figure 5. Pattern formation with Chebyshev points.

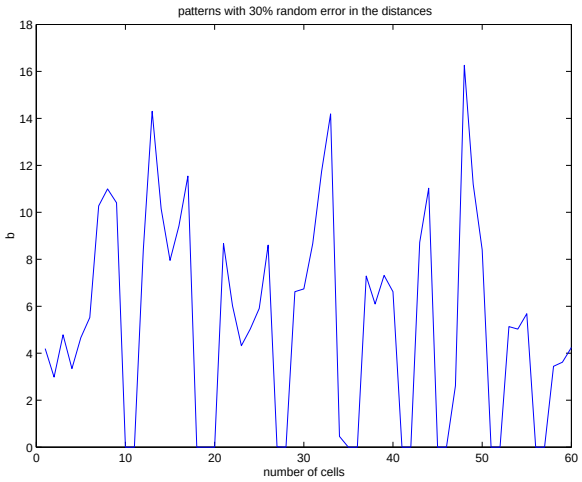

Figure 6. Patterns with 30 percent random error in the distances.

we threshold the morphogen level leads to minimal impact from the shape of the pattern.

4.3 Pattern Formation by Simple Diffusion

In some applications it may be desirable to form linear stripes or rings more simply than by using reaction-diffusion systems. In this case, we propose to use diffusion directly as a pattern formation mechanism. This is achieved as follows:

- diffuse a counter value (modulo n), and
- threshold at k for stripes of width n-k.

This can form circular patterns (from a point source) or linear patterns (from a line source).

5. Conclusions and Future Work

We have discussed here some of the issues related to pattern formation in irregular meshes of S-Nets. Our main result is that patterns can indeed be formed, however, the pattern can vary based on inter-node distances. However, we have determined that such meshes can lead to much more rapid pattern formation.

Some further issues that we hope to resolve in the near future include:

- the diffusion part of the equation is typically computed by a finite difference approximation. Given the sensitive nature of the equations involved, we intend to look at other models of diffusion: e.g., random walk or flux differences. In fact, these may lead to more robust or

efficient computational methods. (See [9] for a good review of diffusion phenomena.)

- Only a few pattern forming systems have been studied in any detail. A wider search for action specific mechanisms may yield a toolkit of algorithms for specific system goals. We also intend to look at other new techniques for pattern formation, such as reported by Peletier and Troy [29] who study a family of fourth order differential equations.

References

[1] A. Babloyantz and J. Hiernaux. Models for cell differentiation and generation of polarity in diffusion-governed morphogenetic fields. *Bulletin of Mathematical Biology*, 37:637–657, 1975.

[2] J. Bard and I. Lauder. How well does turing's theory of morphogenesis work? *Jnl Theor Biology*, 45:501–531, 1974.

[3] P. Bonnet, J. Gehrke, and P. Seshadri. Towards sensor database systems. In *Proc of the Second Intntl Conf on Mobile Data Management*, Hong Kong, January 2001.

[4] N. Bulusu, D. Estrin, L. Girod, and J. Heidemann. Scalable coordination for wireless sensor networks: Self-configuring localization systems. In *Proc. Sixth International Symposium on Communication Theory and Applications (ISCTA '01)*, Ambleside, Lake District, UK, July 2001.

[5] Y. Chen. Snets: Smart sensor networks. Master's thesis, University of Utah, Salt Lake City, Utah, December 2000.

[6] Y. Chen and T. C. Henderson. S-nets: Smart sensor networks. In *Proc International Symposium on Experimental Robotics*, pages 85–94, Hawaii, December 2000.

[7] D. Floreano, J.-D. Nicoud, and F. Mondada, editors. *Advances in Artificial Life, 5th European Conference, ECAL'99, Lausanne, Switzerland, September 13-17, 1999, Proceedings*, volume 1674 of *Lecture Notes in Computer Science*. Springer, 1999.

[8] D. Ganesan, R. Govindan, S. Shenker, and D. Estrin. Highly resilient, energy efficient multipath routing in wireless sensor networks. *Mobile Computing and Communications Review*, 1(2), 2002.

[9] R. Ghez. *Diffusion Phenomena*. Kluwer, New York, 2001.

[10] M. Granero, A. Porati, and D. Zanacca. A bifurcation analysis of pattern formation in a diffusion goverened morphogenetic field. *Jnl of Mathematical Biology*, 4:21–27, 1977.

[11] T. C. Henderson. Leadership protocol for s-nets. In *Proc Multisensor Fusion and Integration*, pages 289–292, Baden-Baden, Germany, August 2001.

[12] T. C. Henderson, M. Dekhil, S. Morris, Y. Chen, and W. B. Thompson. Smart sensor snow. *IEEE Conference on Intelligent Robots and Intelligent Systems*, October 1998.

[13] T. C. Henderson, J.-C. Park, N. Smith, and R. Wright. From motes to java stamps: Smart sensor network testbeds. In *Proc International Conf on Intelligent Robots and Systems*, page to appear, Las Vegas, NV, October 2003.

[14] J. Hill and D. Culler. A wireless embedded sensor architecture for system-level optimization. Ece, UC Berkeley, October 2002.

[15] T. Imielinski and S. Goel. Dataspace - querying and monitoring deeply networked collections in physical space. In *Proc. of International Workshop on Data Engineering for Wireless and Mobile Access (MobiDE'99)*, Seattle, WA, August 1999.

[16] C. Intanagonwiwat, R. Govindan, and D. Estrin. Directed diffusion: A scalable and robust communication paradigm for sensor networks. In *Proc. of Mobicom 2000*, Boston, August 2000.

[17] E. Justh and P. Krishnaprasad. Pattern-forming systems for control of large arrays of actuators. *Jnl of Nonlinear Sci*, 11(4):239–277, 2001.

[18] J. Koenderink. The structure of images. *Biol. Cyber.*, 50:363–370, 1984.

[19] S. Kumar. DARPA sensit program. In S. Kumar, editor, *DARPA Sensit PI*, pages on–line papers, St. Petersburg, FL, 2001.

[20] T. Lacalli and L. Harrison. Turing's conditions and the analysis of morphogenetic models. *Jnl of Theoretical Biology*, 76:419–436, 1979.

[21] A. Lim. Support for reliability in self-organizing sensor network. In *Proc of the Intnl Conf on Information Fusion*, Annapolis, Maryland, July 2002.

[22] S. Madden, M. Franklin, and J. Hellerstein. Tag: a tiny aggregation service for ad-hoc sensor networks. In *Proc. of the Fifth Symposium on Operating Systems Design and Implementation*, Boston, MA, December 2002. USENIX Association.

[23] P. Maini and H. Othmer. *Mathematical Models for Biological Pattern Formation*. Springer-Verlag, Berlin, 2001.

[24] A. Mainwaring, J. Polastre, R. Szewczyk, D. Culler, and J. Anderson. Wireless sensor netwroks for habitat monitoring. In *WSNA 2002*, Atlanta, GA, September 2002.

[25] H. Meinhardt. *Models of Biological Pattern Formation*. Academic Press, London, 1982.

[26] J. Mervis. Science with an agenda: NSF expands centers programs. *Science*, 297:506–509, July 2002.

[27] J. Murray. *Mathematical Biology*. Springer-Verlag, Berlin, 1993.

[28] R. Nagpal. Programmable pattern-formation and scale-independence. In *Proc International Confernce on Complex Systems (ICCS)*, Nashua, NH, June 2002.

[29] L. Peletier and W. Troy. *Spatial Patterns*. Birkhauser, Boston, 2001.

[30] P. Perona, T. Shiota, and J. Malik. Anisotropic diffusion. In B. Romeny, editor, *Geometry-Driven Diffusion in Computer Vision*. Kluwer, 1994.

[31] A. Perrig, R. Szewczyk, V. Wen, D. Culler, and J. D. Tygar. SPINS: Security protocols for sensor networks. *Wireless Networks*, 8(5):521–534, Sept 2002.

[32] V. Swaminathan, K. Chakrabarty, and S. Iyengar. Dynamic i/o power management for hard real-time system. In *Proc. Intl. Symposium on Hardware/Software Co-Design (CODES*, pages pp. 237–242, Ambleside, Lake District, UK, 2001.

[33] A. Turing. The chemical basis of morphogenesis. *Philosophical Transactions of the Royal Society of London*, B237:37–72, 1952.

[34] K. Whitehouse and D. Culler. Calibration as parameter estimation in sensor networks. In *Proc. WSNA 2002*, Atlanta, September 2002.

[35] Y. Yemini, S. da Silva, D. Florissi, and H. Huang. The network flow language: A mark-based approach to active networks. Computer Science XXX, Columbia University, July 1999.

[36] L. Zhang. Simple protocols, complex behavior. In *Proc. IPAM Large-Scale Communication Networks Workshop*, March 2002.