An Interior Ellipsoid Algorithm for Fixed Points*

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Abstract

We consider the problem of approximating fixed points of non-smooth contractive functions with using of the absolute error criterion.

In [12] we proved that the upper bound on the number of function evaluations to compute ε-approximations is $\mathrm{O}(n^3(\ln\frac{1}{\varepsilon}+\ln\frac{1}{1-q}+\ln n))$ in the worst case, where $0 < q < 1$ is the contraction factor and n is the dimension of the problem. This upper bound is achieved by the circumscribed ellipsoid (CE) algorithm combined with a dimensional deflation process.

In this paper we present an inscribed ellipsoid (IE) algorithm that enjoys $\mathrm{O}(n^2(\ln\frac{1}{\varepsilon} + \ln\frac{1}{1-q} + \ln n))$ bound. Therefore the IE algorithm has almost the same (modulo multiplicative constant) number of function evaluations as the (nonconstructive) centroid method [11]. We conjecture that this bound is the best possible for mildly contractive functions ($q \approx 1$) in moderate dimensional case. Affirmative solution of this conjecture would imply that the IE algorithm and the centroid algorithms are almost optimal in the worst case. In particular they are much faster than the simple iteration method, that requires $\left\lceil\frac{\ln(1/\varepsilon)}{\ln(1/q)}\right\rceil$ function evaluations to solve the problem.

Key words: Fixed points, inscribed ellipsoid algorithm, optimal complexity algorithm.

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