Analysis of Tolerance for Manufacturing Geometric Objects from Sense Data

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Abstract

In this work we address the problem of manufacturing machine parts from sense data. Constructing geometric models for the objects from sense data is the intermediate step in a reverse engineering manufacturing system. Sensors are usually inaccurate, providing uncertain sense information. We construct geometric entities with uncertainty models for the objects under consideration from noisy measurements and proceed to do reasoning on the uncertain geometries, thus, adding robustness to the construction of geometries from sensed data.

1 Introduction

Reverse engineering is a process that reconstruct a representation of a physical model, so that it can be reproduced identically. It is a new branch in the CAD/CAM field. Parts are manufactured according to blue prints, but when blue prints are not available, (such as, the part is too old, and its blue prints are missing), reverse engineering can be used to reproduce these parts. This can be achieved by the following two steps: sensing the part to construct its CAD representation and then manufacturing the part according to the representation. It is easy to see that the accuracy of measurement is the key to success in reproducing an accurate CAD model.

The accuracy of the measurement can be improved not only by improving the quality of measuring instrument, but also by optimizing sampling data. A reverse engineering system has been built and the measuring process is done by a vision sensor (B/W CCD camera) and a coordinate measuring machine (CMM). The physical model is inspected by cooperating the observer camera and the probing CMM. The observer camera provides a high level (qualitative) description of the physical model, and the CMM complete the CAD model with precise parametric data. Figures 1 and 2 provide an over view of the whole system. Figures 3, 4, and 5 show original and reverse-engineered CAD models and physical parts. Figure 6 shows the vision setup.

In order to increase accuracy and efficiency of the measurement, a feedback sensing system is designed as shown in figure 7.

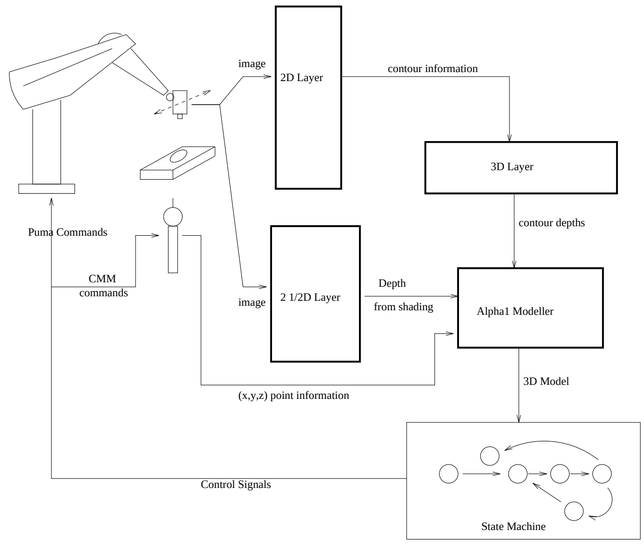

Figure 1: Overview of the system

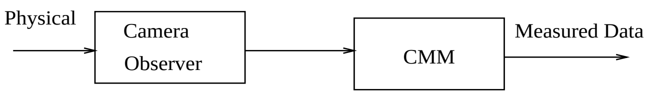

Figure 2: Sensing

In this feedback system, a probabilistic geometric modeler is involved as a feedback agent to provide information for further measurements required to refine the CAD model, and also gives a quantitative measure of the accuracy of the current CAD model. The CMM machine actively measures the parameters for lo-

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cal features. By using the probabilistic geometric modeler performing geometric modeling operations, redundant information of the part geometry will be computed to reduce the load of the CMM measurement activities. Therefore, it improves the efficiency of the sensing process, besides, the geometric reasoning on the probabilities of uncertain geometries can guide the CMM to perform focused measurements to allow for higher accuracy and efficiency. For instance, the slot (see figure 8) in mechanical engineering is a commonly used feature, and the parallelism of the two side planes is an important measurement.

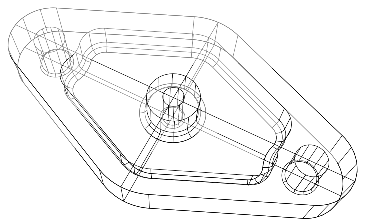

Figure 3: CAD Model of the Original Part

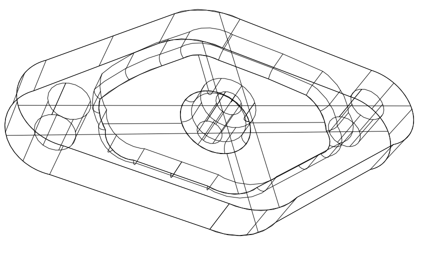

Figure 4: CAD Model of the Reverse-engineered part

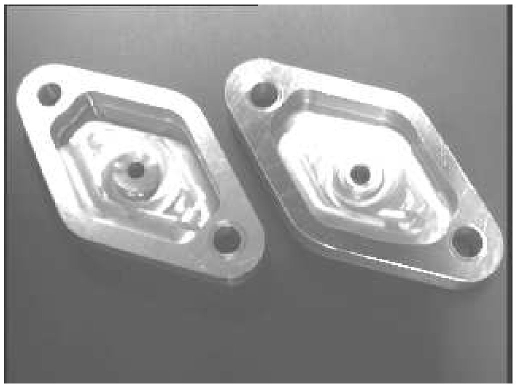

Figure 5: Original and Reverse-engineered Parts

The two side planes are based on sampling points from CMM and/or visual data. Measurements of these points are not exact, therefore, these two planes that are constructed from these measurements, are planes with probabilities as the confidence measure. Consequently, the parallelism is no longer a definite relation, it has a probability distribution. If the confidence of the parallelism does not satisfy the manufacturing requirement, refinement of the two side planes is required, hence remeasuring of the points is performed.

Figure 6: Experimental Setup

Some work has been done in the probabilistic relationship between the geometric objects and their relations, but the probability relations between the sampling points and geometric primitives have not yet been studied extensively. The geometric objects that this probabilistic geometric modeler is based on are constructed from sensing data. Therefore, study of the relation of the probabilistic characters of geometric objects and sensing data is very important. This paper presents the study of these relations. The work ad-

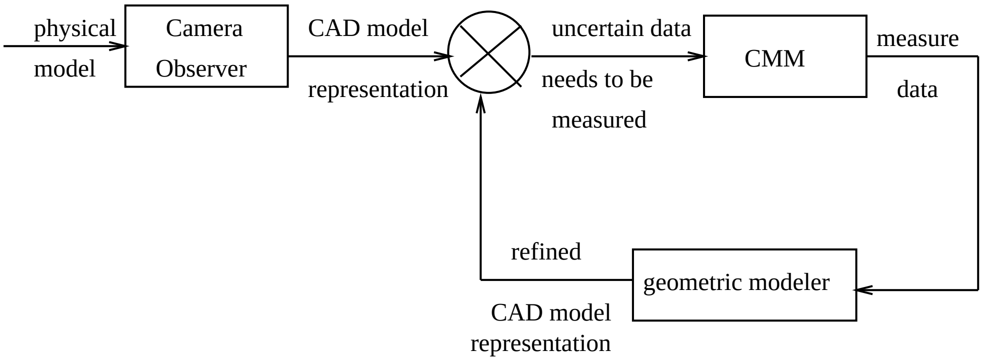

Figure 7: Feedback Sensing System

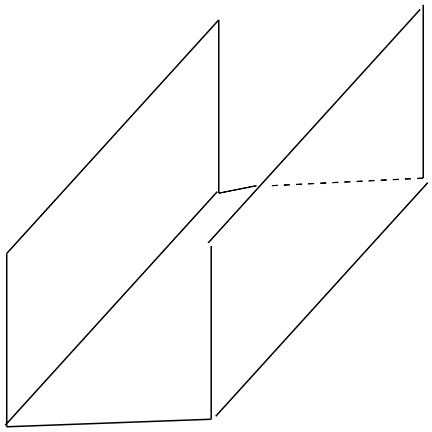

Figure 8: Slot

dresses the statistic geometric objects constructed from sensing data, relations of these statistic geometries, and the effect of decisions on its relative geometric objects.

2 Related Work

Stochastic geometry has been systematically studied by mathematicians. In [12], mathematical theories of stochastic geometry are well studied, and uncertain geometric features can be represented as constrained functions. Classic examples of stochastic geometry can be found in [11]. Kendall and Moran[12], describe a method of choosing distributions on geometric elements which provide a consistent interpretation of physical geometric elements.

Recently, research about sensing and uncertain geometry in robotics presents lots of ideas for handling uncertainty geometry. Hugh F. Durrant-Whyte in [5, 4] has modeled the sensor in a manner that explicitly accounts for the inherent uncertainty encountered in robot operations. In Davidson's thesis[13], he made the important observation that arbitrary random geometric objects can be described by a point process in parameter space.

In computer-aided geometric modeling, methodologies for building a robust geometric modeler explores ways of handling the uncertain geometry caused by the imprecise computations. Arbitrary decisions are made, when uncertainty arises. In [14, 2, 8, 7, 3, 1, 9, 6, 10], three region tolerances are used to keep track of uncertainty caused by the computational error. In [15], arbitrary decisions are made and corresponding uncertainties are restricted.

3 Representations for Uncertain Geometry

In geometric modeling, algorithms and representations for geometric objects are well developed, but the tolerance (uncertainty of geometry) has not yet been well defined. In [14], a geometric object is represented by boundary and hybrid representations, associated with a tolerance representing the uncertainty of the geometry. Figures 9 to 13 presents tolerancing for some well known geometric features.

Based on the representations that has been developed and used in [14], a representation for uncertain geometry is developed as follows:

An uncertain geometric object is represented in two parts: a geometric description, and a probabilistic distribution of geometry. The geometric description is a parameter vector, and the probabilistic distribution of geometry is a vector of the same dimensions as the geometric description, but with corresponding probabilistic distributions of the parameters.

For instance, a plane can be specified as a equation: $(A, B, C), (f_a, f_b, f_c)$, where (A, B, C) is the geometric description and $z = Ax + By + C$. (f_a, f_b, f_c) is the probabilistic distribution of geometry, and also can be specified in another form: $(P, V), (f_p, f_v)$, as shown in figure 14, where P is a base point, and V is the normal vector of the plane. f_p is the uncertainty of the base point, and f_v is the uncertainty of the normal vector. It can be proved that f_p and f_v can be computed

Figure 9: empty

Figure 10: empty

Figure 11: empty

Figure 12: empty

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from f_a, f_b, f_c, and P, V can be computed from (A, B, C). By defining f_a, f_b, f_c, different types of probability distributions can be handled by this representation.

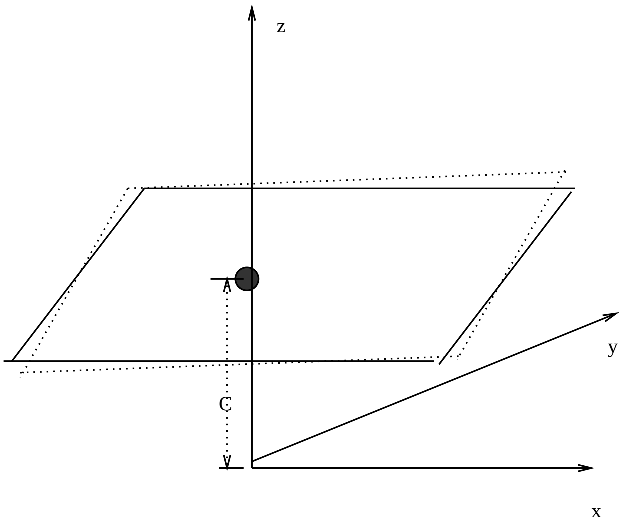

Figure 14: Representation of a Plane

4 Experiment on Statistic Geometric Objects Constructed From Sensing Data

The geometric objects that the modeler operates on, are constructed from the sensing data. How the distribution of sensing data affects the uncertainty of the geometry is the basis for defining the distributions of the geometry. In this section, the uncertainty of the plane relating to the sensing 3D coordinates is studied. A set of discrete sensing data is used to perform the computations.

4.1 Best Least Square Fit

In order to reduce the random error, usually, n sampling points are measured to defining a plane, yet the points have certain probability distributions which mainly depend on the measuring machine (e.g CMM), the n points are independent random events. Therefore, a best least square fit method for computing the plane parameter is used. This approach gives the maximum liklehood result, and confidence on the sampling data to be a plane.

Assuming that input data is (x_i, y_i, z_i), where x_i, y_i, z_i are either fixed values, or probability functions. They can be either independent, or correlated. Explicit function definition for a plane in 3D will be $z = Ax+By+C$. If there are n points, the best least fit plane should be the solution of the following equation set.

$$Z = P \bullet X$$

Where

$$Z = \begin{bmatrix} z_1 \\ z_2 \\ \dots \\ z_n \end{bmatrix}$$

$$P = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ \dots & & \\ x_n & y_n & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

Because P is an n x 3 matrix, and X is a 3 x 1 matrix, $rank(P) = 3$, and $n \geq 3$, solution of X is unique. When $n > 3$, the solution X is a best least square fit.

$X = (P^T \bullet P)^{-1} \bullet P \bullet Z$

Or in the other form:

$$\begin{aligned} A &= f(x, y, z) \\ B &= g(x, y, z) \\ C &= h(x, y, z) \end{aligned}$$

Where $x \in [x_1, x_2]$, $y \in [y_1, y_2]$, $z \in [z_1, z_2]$ are discrete. Function f, g, h are non-linear functions. To compute the probability distribution of A, B, and C, exhaustively computing values of f, g, and h, will provide the discrete probability distribution array for A, B, and C.

From the above mathematics, we can see that the computation complexity is exponential. If m is the number of distribution values and n is the number of sampling points, this above computation will be performed $(3^m)^n$ times.

4.2 Sensing Data and its Corresponding Results

The sensing data is modeled by discrete points with their corresponding probabilities. Normally, a point in 3D is represented as (x, y, z), but for this sensing data, x, y, and z, are no long a single value, they are distributions as shown in figure 15.

Due to the computational complexity, and the generality of the problem, a three distribution values data set is used for experiments. The resulting planes (A, B, C) along with their distributions are computed. Graphs of A,B, and C distributions are approximated by the following computations.

What we want to get is the concept of the $f(x)$ shape. The data we computed are discrete state vector (A, B, C) and its probability. $P(x_i < x < x_{i+1}) = \int_{x_i}^{x_{i+1}} f(x)$ is computed and plotted, where x can be A, B, or C. and $x_{min} \leq x_i \leq x_{max}$. In order to smooth the curve, an overlapped set of x_i is used. In the result

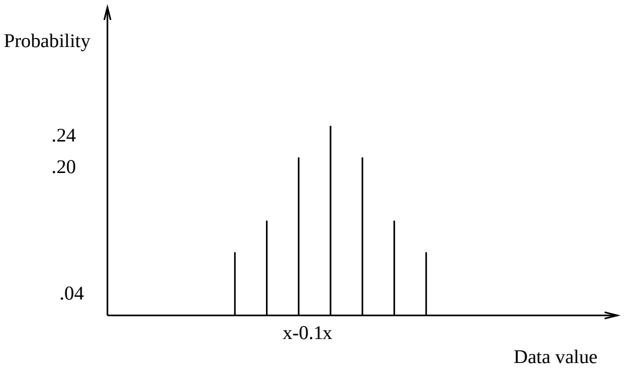

Figure 15: Sensing Data

figures, the x axis are the values of A, B, C respectively, and the y axis are the corresponding probability of that value.

Test 1: Uniform distribution: the sensing data is shown in figure 16.

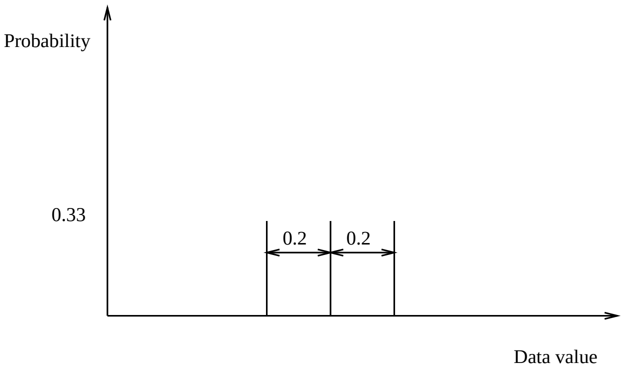

Figure 16: Uniform Distribution

There are a total three points with such distributions, planes defined by these points are computed. The distributions of A, B, C are shown in the following figures.

Test 2: Gaussian distribution: the sensing data is shown in figure 17

There are a total of three points with such distributions, the planes defined by these points are computed. The distributions of A, B, C are shown in figures 18 to 20.

From the uniform distribution and gaussian distribution test data, we can see that the distribution of the (a, b, c) space is Gaussian despite of the probability distributions of the sensing data.

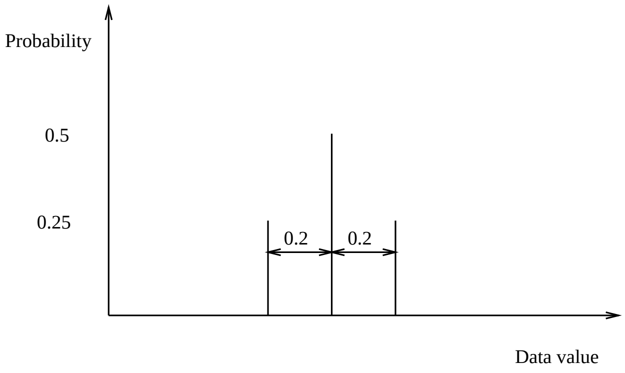

Figure 17: Gaussian Distribution

5 Relations of Statistic Geometries and its Effect on Relative Geometries

As mentioned in the introduction, the goal of this probabilistic modeler is to feedback control the sensing devices to measure the physical model and give a quantitative confidence measurement for the CAD model. Some relations of these uncertain geometries are computed, and results are computed with their uncertainty distributions.

Basically, geometric relations are set relations, such as: intersecting, coincidence, incidence, apartness, and parallelism. Because of the uncertainty of the geometries, these relations are not definite, they are decisions with certain confidence, also, this confidence can be specified by its probability. For instance, a point incident on a plane, can be computed as a point incident on the plane with 0.9 probability. This provides reasoning based on probabilities.

A feedback computation of a plane that is supposed to be collinear with a given plane is studied. A program that takes the output discrete planes along with their probabilities is implemented, and the cases of parallel and collinear statements are computed with their probabilities. Some examples of parallelism and collinearity have been tested. For example, collinearity and parallelism of the uniform distribution planes (as described above has been tested). The probability for parallelism is 0.824719, for collinearity is 0.334722. The parallelism and collinearity of the planes of the three points Gaussian distribution and the uniform distributions have also been tested. The parallelism is 0.66730846, and the collinearity is 0.27099140. (the tolerance for testing them is the square distance less than $10e^{-2}$).

If we assume that the plane constructed from the uniform distribution sensing data is decided to be collinear to the plane defined by the above table, then, its distribution is recomputed as follows: among this plane set, the plane instances which are not collinear with any of the plane instances in the given plane set, is discarded.

Figure 18: empty

Figure 19: empty

Figure 20: empty

Figure 21: empty

A	B	C	P(probability)
1.034723	-0.961805	2.386458	0.33
1.036584	-0.966461	2.391768	0.34
1.038042	-0.970109	2.395926	0.33

After discarding these plane instance, the distribution of the new plane set is re-normalized. The resulting distributions of A, B, C is shown in Figure 21

We can see that after recomputing the plane, the distributions of A, B and C are located in a more narrow range, further more, based on this redistributed plane set, the sampling points can also be recomputed and some of the sampling points can be discarded, or a remeasurement of these points is performed.

6 Conclusions

Based on real sensing data, the probability of the geometry of the objects under consideration is computed. This provides us with the capability to define the probability distribution of the geometry based on robust computations as opposed to noisy measuring instruments. The relations between uncertain geometries are dependent on the uncertainty of geometries. Quantitative measurement for the constructed CAD model can thus be computed, and the relation can also involve the redistribution of the uncertainty of the geometry, this can be used as a feedback to guide the sensing and manufacturing modules.

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