Constraint Objects – Integrating Constraint Definition and Graphical Interaction

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Abstract: This paper describes the implementation of a new constraint-based technique for direct manipulation in interactive CAD, which will simplify the design process, especially in the early stages. We introduce so called Constraint Objects and Parameter Objects which constitute an object-oriented view on constraints. They serve to simulate the mutual degrees of freedom between objects for which a geometric relation (distance, angle, parallel, congruence, etc.) has been defined. A 2-D profile editor has been realized for interactively constructing lines and circles in various ways. Each construction operation, implicitly defines constraints to capture the intent of the operation. These constraints are represented by corresponding parameter objects and constraint objects. A constraint solver is applied to rewrite the set of constraints into its normal form if necessary. Finally, the resulting parameter-constraint-regular-object network serves to simulate the degrees of freedom of geometric objects during interactive dragging manipulations, and to make sure the existing constraints are not violated by subsequent operations.

1 Introduction

Conventional modeling systems do not support the free dimensioning of geometric objects by means of constraints, but require users to construct them by a sequence of geometric operations. Mechanical parts designed by such a CAD system are represented as fixed geometry; the geometric design part is completely separated from other design criteria. It is difficult for a user to add information under a different view, later on. Changing a part may inadvertently violate previous design decisions.

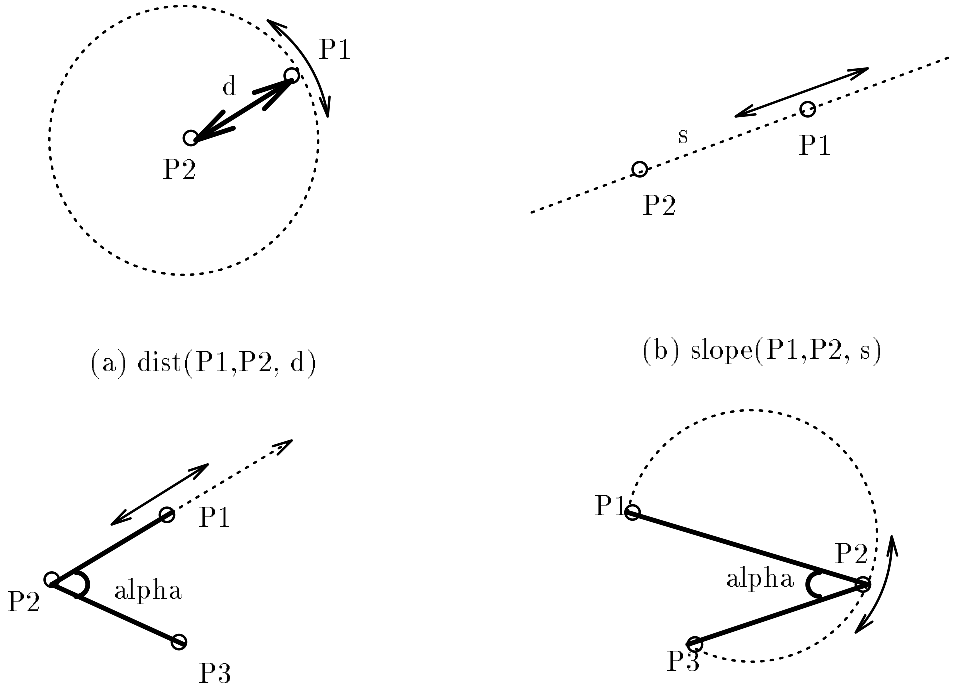
Geometric constraints have proven useful for interactive geometric design (see bibliography for references). The idea is to specify shape by constraints such as distances, angles, etc. and use a constraint solver to derive the shape from such a specification. A clear drawback of a constraint based approach is that it is not intuitive for complex examples. It is extremely difficult for a designer to come up with a complete and consistent set of constraints. Often we encounter over and under specified parts simultaneously that are hard to resolve in a specification. This is addressed, e.g. in [14]. Also constraint solving is very difficult, even if the specification is consistent. Most constraint based systems use numerical techniques (relaxation, Newton iteration) which can theoretically solve problems that don't have a closed form algebraic or geometric solution; on the other hand numerical techniques have convergence problems that make them very unpredictable. We investigated ways of specifying geometric objects by geometric constraints, and developed a new mechanism for symbolic geometric constraint solving. Constraints are represented symbolically as predicates over points. A rewrite rule mechanism seeks to match the left hand side of a rule with a subset of the constraints. If a rule applies some of the predicates are replaced by new, simpler ones, and a construction operation is applied to satisfy these constraints simultaneously [7], [8]. In [21], [22] we have shown that object-oriented graphical interaction can be integrated naturally with constraint definition and constraint solving for 3-d assembling of mechanical parts. Other approaches that integrate constraint definition and interactive modeling can be found, for instance in [10], [13], [17], [18], [19], and [23]. Constraints can also be used to communicate unfinished design, etc. (constraints are part of the model data). This solves in part the problem of later modifications being consistent with previous ones, allowing one to communicate ideas between different departments of the design office during the early design process. In this paper we continue our efforts to integrate constraint solving techniques with graphical object-oriented interaction by introducing so called constraint objects.

2 Constraint Objects

The role of the constraint objects is to simulate relative degrees of freedom between constrained objects. The types of constraint objects supported in here are chosen to be compatible with the types of constraints used in the symbolic geometric constraint solver, described in [8].

- pos(A, Pos): The position of point A by a symbolic expression Pos.
- dist(A,B, d): The distance between points A and B is defined by d.
- slope(A,B, s): The slope of the line through A and B is s.
- vector(A,B, v): Point B is offset from point A by a vector v.
- angle(A,B,C, Alpha): The angle between line AB and line BC is Alpha.

In the following sections the function of the constraint objects is described in some detail.



(a) dist(P1,P2, d)

(b) slope(P1,P2, s)

(c) angle(P1,P2,P3, alpha)

(d) angle(P1,P2,P3, alpha)

Figure 1: The various degrees of freedom for different constraint objects

2.1 The Effect of an Individual Constraint

A point that is constrained by one of the above constraints has a degree of freedom determined by the type of constraint and the degrees of freedom of the other points constrained by the same constraint.

Figure 1 shows various cases for the degree of freedom of a point related to other points by one constraint, assuming that the other points remain fixed. For the constraints introduced here, the loci defined by a constraint object are circles and straight lines.

Table 2.1 determines the degrees of freedoms of a point attached to one constraint, assuming that one other point attached to the same constraint has a degree of freedom of 2, 1, or zero, respectively.

constraint type	o.p. has dof 0	o.p. has dof 1	o.p. has dof 2
position	0	na	na
vector	0	1	2
slope	1	2	≥ 2
angle	1	2	≥ 2
distance	1	2	≥ 2

Table 1: the degree of freedom of a point to which a constraint has been attached (assuming that another point attached to that constraint has 0, 1, or 2 dofs)

intersecting two constraints	resulting dof
$2 \wedge 2$	2
$2 \wedge 1$	1
$2 \wedge 0$	0
$1 \wedge 1$	0
$1 \wedge 0$	0
$0 \wedge 0$	0

Table 2: multiple constraints attached to one point with different dofs; here's how they add

2.2 Multiple Constraints

Multiple constraints attached to the same point further restrict this point's degree of freedom. Table 2.2 shows the effect of two constraints attached to the same point each determining a degree of freedom, taken individually.

From this information, we are now going to derive a recursive algorithm, that can determine for any point P that is directly or indirectly constrained to other points by many constraints, what its degree of freedom is, and which other points need to be changed, when P is changed within its degree of freedom. The way the function is used, is that one requests a degree of freedom for a point P, by calling rqdofP(P, dof), where dof = 2, 1, or 0. The function then checks whether that can be achieved, and which other points are involved, and how. In 2-D we can start requesting a degree of freedom of 2, i.e. we want to move the point freely, but other points have to react to the changes. If there is no way that point P can be moved freely, we can try to request a degree of freedom of 1. If successful the point can be moved along a curve. If it fails, the point cannot be moved at all.

The algorithm is subdivided in 2 recursive functions, mutually calling each other.

```
procedure rqdofP(P, dof)  (*requests a degree of freedom for a point *)
  begin
```

```
    find the set C of constraints attached to P
    if dof = 2 then  (* all the dofs requested from all the
                        constraints attached to P must be 2 *)
      for each constraint C1 in C do
        rqdofC(P, 2, C1)

    if dof = 1 then
      begin
        pick some constraint C1 and call rqdofC(P, 1, C1)
        for each constraint C2 in C where C2 != C1 do
          rqdofC(P, 2, C2);
      end
    if dof = 0 then return;
  end

------------------------------------------------

Procedure rqdofC(P, dof, Constr)
(* requests a degree of freedom from a single constraint *)
  begin
    if dof = 2 then
        IF Constr = pos(P,_) then fail;
        IF Constr = vector(P,P1,_)
           or
           Constr = vector(P1,P,_) then rqdofP(P1, 2);
        IF Constr = slope(P,P1,_)
           or
           Constr = slope(P1,P,_)  then rqdofP(P1, 1);
        IF Constr = dist(P,P1,_),
           or
           Constr = dist(P1,P,_)   then rqdofP(P1, 1)
        IF Constr = angle(P,P1,P2,_),
           or
           Constr = angle(P1,P,P2,_),
           or
           Constr = angle(P1,P2,P,_)
 then rqdofP(P1, 1); rqdofP(P1, 0)
                                      or
      rqdofP(P2, 1); rqdofP(P2, 0)
    else if dof = 1 then
        IF Constr = pos(P,_) then fail;
        IF Constr = vector(P,P1,_)
           or
```

```
         Constr = vector(P1,P,_) then rqdofP(P1, 1);
      IF Constr = slope(P,P1,_)
         or
         Constr = slope(P1,P,_)  then rqdofP(P1, 0);
      IF Constr = dist(P,P1,_),
         or
         Constr = dist(P1,P,_)   then rqdofP(P1, 0)
      IF Constr = angle(P,P1,P2,_),
         or
         Constr = angle(P1,P,P2,_),
         or
         Constr = angle(P1,P2,P,_) then rqdofP(P1, 0); rqdofP(P1, 0)
                                   or
                                   rqdofP(P2, 0); rqdofP(P2, 0)
  else if dof = 0 then
         always succeed
end
```

3 Evaluating the Degrees of Freedom

3.1 Phase 1: Finding the dependency graph

To illustrate the effect of the algorithms we can depict the relationships by directed, acyclic hyper-graphs. A node in the graph shows a point together with its degrees of freedoms. An arc in the graph represents a constraint and is labeled with the degrees of freedoms associated with the constraint. Looking at it in another way, a node corresponds to a call of rqdofP(P, dof), and an arc to a call rqdofC(P, dof, C). When we try to move a point, a dependency graph is established this way. An example is shown in Figure 2.

For a certain constraint network, there might be several valid dependency graphs associated with it. This is reflected by the non-deterministic formulation of the algorithms, using logical 'or' between statements.

We therefore need to have a criterion for selecting an 'optimal' graph among them. Ideally, we will expect the move to affect as few other points as possible. Or in terms of the graph, we would like to minimize the levels of the graph. As a consequence, the search strategy we adopt is breadth-first search together with backtracking.

3.2 Phase 2: Evaluating the geometric solution

After determining the degree of freedom of a point we need to evaluate the amount of change in the positions of the points involved that need to be adjusted. The design of this algorithm

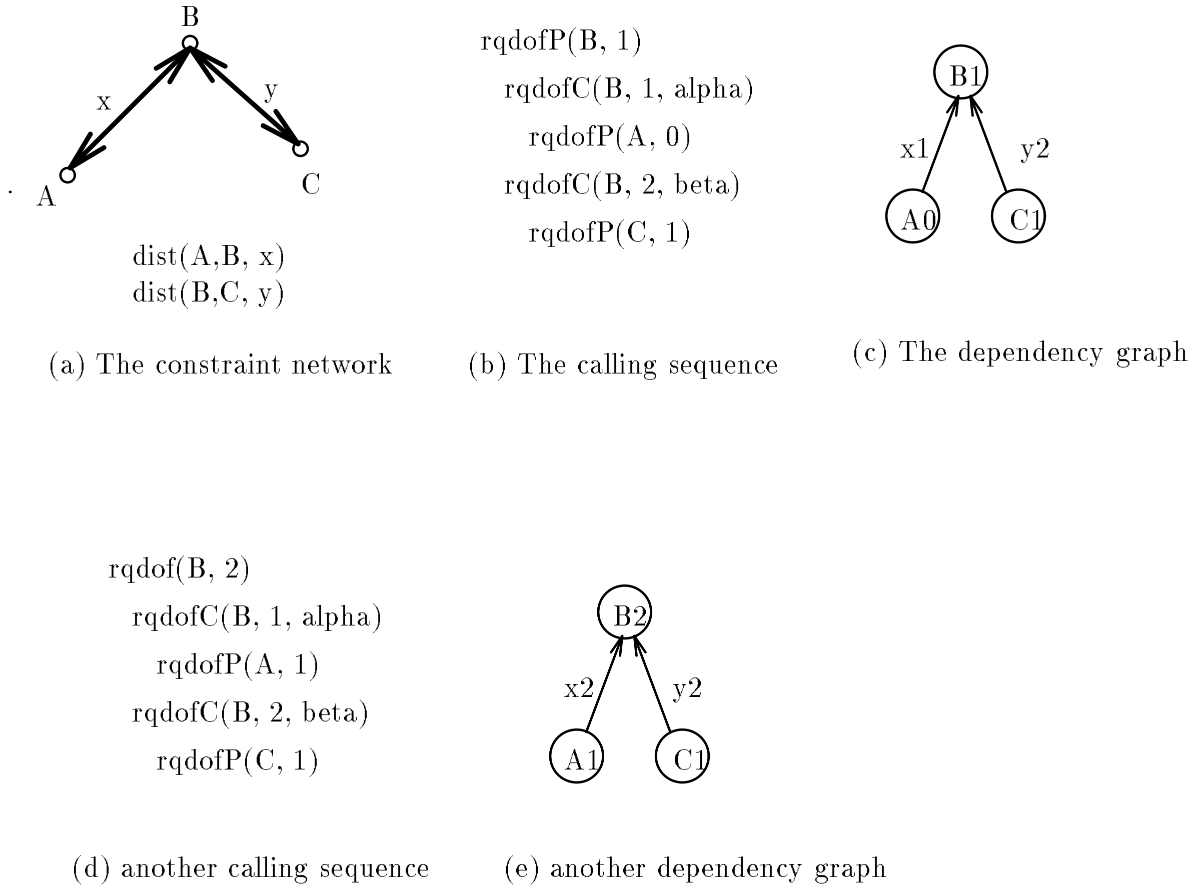


Figure 2: The constraint network and its dependency graph

emphasize on efficiency so that user can get fast feedback from display. As a consequence, iterative methods are deliberately avoided.

We realize that it is not possible for the user to predict where to drag the point when it has one degree of freedom. Naturally, we the user only specifies the amount and the direction of the change rather than the point following the cursor.

Note also that once the dependency graph is established, we don't need to construct the graph again each time we try find a new position. Instead, we can use this graph to evaluate the new dimensioning of the network as many times as we want. However, there is no guarantee that a geometric solution will exist even if a dependency graph does exist. For instance, when one tries to drag the point too far or in an impossible direction. In these case, this algorithm will do nothing to the points so that the constraints are still maintained.

This algorithm uses the graph produced by phase 1 and determines the position of the constrained points in the graph by a set of rules which governs the order and the means of the evaluation. Depending on the purposes, the rules are further divided into three categories, namely seeding rules, propagation rules, and evaluation rules.

3.2.1 Seeding rules

The seeding rules to some extent determine the order in which the points need to be changed. We have two rules used respectively for one and two degrees of freedom we request from the point changed.

- rule 1: One degree of freedom

  In the simplest cases, the locus of the point dragged will only depend on the constraint which we request one degree of freedom from, ie. all the other points confined by the same constraint remain unchanged, then we can start with this point. On the other hand, if some of the other points constrained by the same constraint request some degrees of freedom to facilitate the change, we need to recurse down the arc to find the position of those points first.

- rule 2: Two degrees of freedom

  If we request two degrees of freedom from the point we dragged the new position is not depending on updates further down in the dependency graph, so we can directly update this point, and then recurse.

3.2.2 Propagation rules

The propagation rules determine how the changes are propagated up in the dependency graph, once they are made. The basic principle behind propagation rules is that a one degree of freedom intersecting with another one degree of freedom yields zero degree of freedom. Consequently, if a point satisfies one of the following conditions, it can fire, and the position can be evaluated.

- condition 1: a point is constrained by a set of constraints and at least two of them give one degree of freedom (ie, the other constraints yield two degrees of freedom).
- condition 2: a point is constrained by only one constraint which yields one degree of freedom.

Once the position of a point is determined, it has only zero degrees of freedom. We need to update the graph to reflect this change. We will first mark the point as having zero degree of freedom and then mark all the constraints attached as having one less degree of freedom.

Note that condition 1 allows points with more than two constraints each yielding one degree of freedom to fire.

3.2.3 Evaluation rules

Depending on the condition which causes a point to fire, we have different evaluation rules. If the point fires by the first seeding rule, the new position of the point will be function of the constraint giving one degree of freedom and a vector propagated from the root. If we request one degree of freedom from the point to be dragged, we can interpret the graph as an equation which gives the locus of the point dragged. In order to solve this simultaneous systems of constraints, therefore, we will have to provide one more equation. In an interactive environment, we normally expected the magnitude and direction of the change correspond to the magnitude and direction of the dragging motion. In other words, we want the new position of the point to be a function of how much and in which direction the user drags the point.

If the point fires by condition 1, we can evaluate the position of the point by finding the intersection of the loci determined by those constraints which give one degree freedom. For example, if a point is constrained by a distance constraint giving one degree of freedom, then the locus of the point is a circle with the center being the other point constrained by the same distance constraint and radius being the distance between the two points. Therefore if a point is constrained by two distance constraints each giving one degree of freedom, we can evaluate the position of the by intersecting two circles. Sometimes, there will be more than one solution. In this circumstance, we need to pick one position among them by some criterion, for example, the point which is nearest to the old position. Other times, there will be no intersection at all and a solution is not possible although phase one produces a valid graph. Therefore the algorithm will fail keeping all the points involved unchanged.

On the other hand, if the point has only one constraint attached, we know that the point will be determined by the locus determined by the constraint. But we still need to pick one point. We propose two methods to pick the point. The first method is to use the old position as a reference, and then find the point on the locus which is nearest to the old position. Alternatively, we can propagate the vector of change of the known point as a reference. If a point has only one constraint, it can use this vector to evaluate its new position.

4 Changing the Parameter of a Constraint

We can extend the previous algorithm to handle the case when user picks a constraint and specifies a new parameter for that constraint. Under these circumstances, we first remove temporarily the constraint from the constraint network, and then request degree of freedom one from one point, and also fix the other points constrained by the constraint. Next, we construct the dependency graph as before. We will then try to find a new solution for the graph in such a way that the newly specified parameter of the said constraint is met. In order to achieve this, we need an iterative method which will take an initial guess on the seeding point, propagate through the whole graph, calculate the offset between the new value and the specified value for the said constraint, and then repeat this process until the offset is under a threshold.

5 Parameter Objects

The idea of constraint objects has been extended to also facilitate congruence relations, parallelism, encidence, etc. In our system we realize such relations by introducing so called parameter objects. In short, a constraint object, for instance a distance constraint, does not carry the value of the distance itself, but instead refers to a parameter object. This way, two distance constraint objects can share the same parameter object. A Boolean flag associated with the parameter object indicates whether this value is fixed (meaning that the constraint object is a distance constraint) or variable (meaning that two or more distances are congruent) and just signifies the current value.

The algorithms described above which determine the degree of freedom of points, and the ones that evaluate the changes during dragging operations, need to be modified to allow for this extra degree of freedom each constraint object may have. During dragging operations the parameter value may be updated when an additional degree of freedom is requested from a constraint object. For all other constraint objects this value is then considered fixed, so that the constraint relation is maintained.

Parallel lines, or collinear lines, for instance, may be realized by two slope constraints sharing the same variable parameter. Coincident points are realized by two position constraint objects sharing the same parameter.

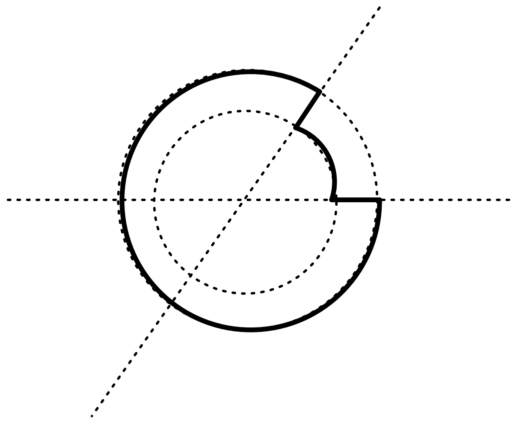


Figure 3: A profile created with the editor

6 A Profile Editor with Implicit and Explicit Constraints

To provide a comprehensive example of the integration of graphical interaction, constraint solving and constraint objects we started implementing a 2-d profile editor. In this implementation 2-d profiles are defined by circle and line segments. Infinite lines and full circles can be defined as references, to simplify interaction. 2-d profiles can be defined by concatenating circle and line segments enclosing an area. Figure 3 shows an

example of a profile defined by intersecting lines and circles, creating lines tangent to circles, parallel or perpendicular to other lines, or as angle bisectors, etc. For every construction operation we automatically create implicit constraints that maintain the characteristics of the operation under later changes.

For instance, a line constructed to be tangent to two circles needs to remain tangent when the radius or position of the circles is changed, or the line itself is changed. Also, explicit constraints can be defined to specify distances, angles, etc. Whenever an explicit constraint is added the constraint solver is run to check its validity and to satisfy simultaneous constraints. The constraint solver itself is based on geometric rewrite rules that can simplify the constraint graph. This paper does not describe the symbolic geometric constraint solver, instead, we refer to [8]. All the implicit and explicit constraints are represented as constraint objects, as described above. Objects can be manipulated directly by dragging them within their degree of freedom specified by the attached constraint objects.

In the example shown in Figure 4 (a), the two circles and the tangent line are created by construction operations which in turn define implicit constraints, ie. the two (right-) angle constraints, to capture the intent of the operation as shown in Figure 4 (b). We later fix the centers of the two circles as well as the radius of the left circle by attaching explicit constraints to them (which are depicted as solid points and solid circle, respectively). Suppose now we try to change the tangent line by dragging point C. The first phase of the algorithm will produce a dependency graph as shown in Figure 4 (d). Then using the seeding rule, point B is picked as the first point to evaluate. From the graph, point B is constrained by the distance constraint *dist*, and therefore it will be on the circle with point A as the center and the distance between point A and point B as the radius. We then determine the new position of point B corresponding to the magnitude and the direction of the dragging motion. Once the new position point B is obtained, point C can be unique determined by the two angle constraints. Figure 4 (e) show one instance of the evaluation phase. Figure 5 shows another example where a line segment with fixed length is attached to two lines with fixed slope.

7 Conclusion

It is crucial to give adequate feedback to the user in the early design phases, when most parts of a design are not yet fully constrained. Constraint objects allow for a uniform, object-oriented representation of constraints by graphical objects which allows for an interactive simulation of under constrained objects and their internal degrees of freedom.

Generally speaking, the capability to specify constraints in an iterative CAD makes manipulation and modification much easier because system takes most of the responsibilities to maintain consistency. In addition, we provide the capability to simulate the degrees of freedom of under-constrained networks of constraints which enable user to draft in a less restricted way in the early design stage.

With constraint objects we completed the framework for interactive geometric modeling that integrates object-oriented manipulation of objects, declarative (relational) definition by constraints (using a constraint solver) and procedural definition (using geometric construction operations).

8 Acknowledgments

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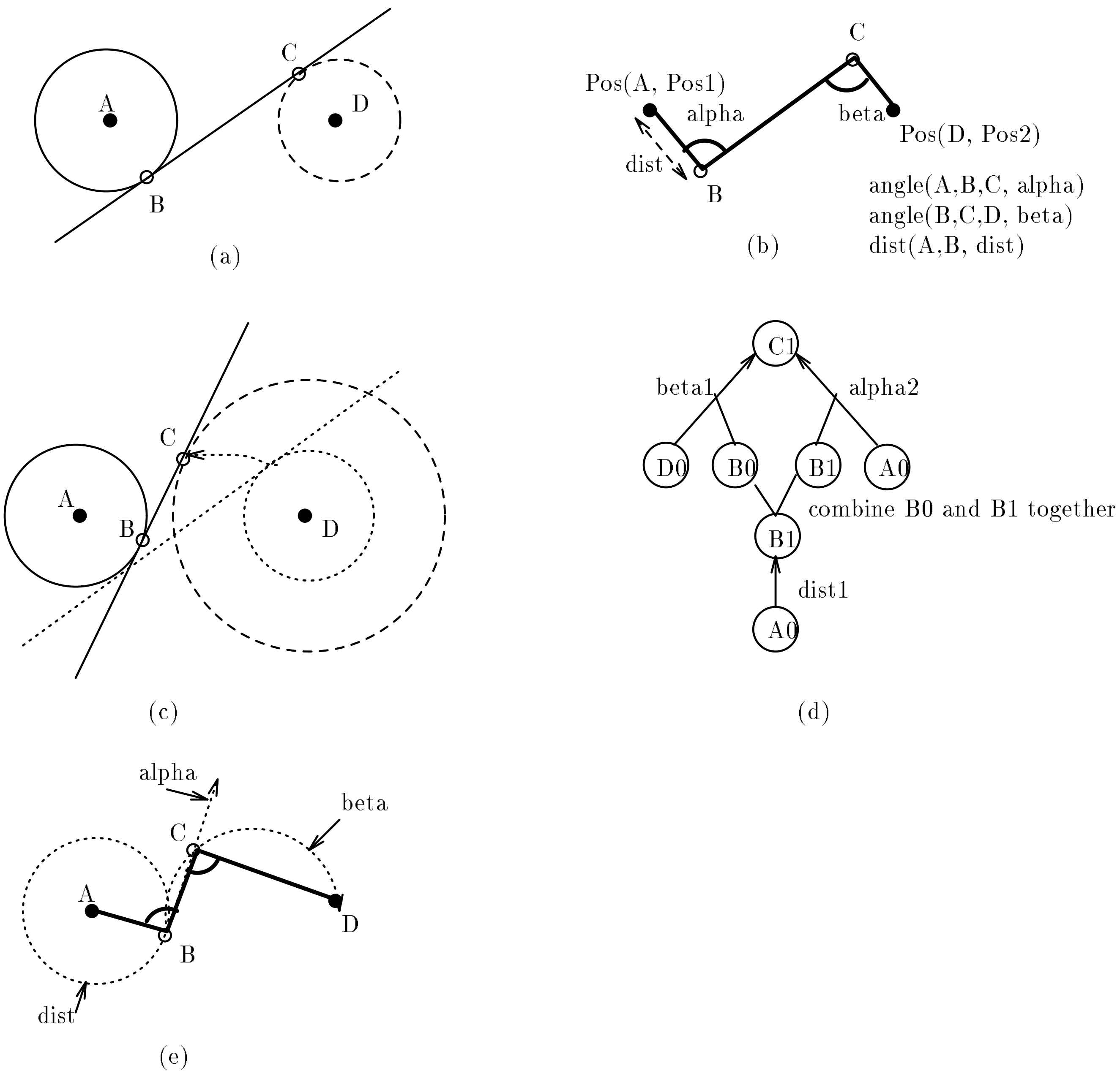

Figure 4: A line tangent to two circles

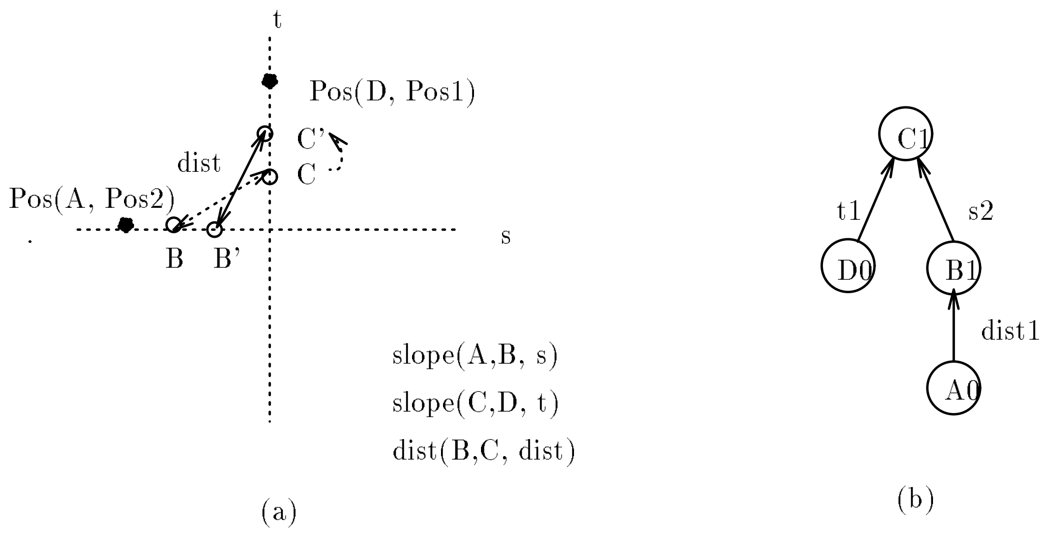

Figure 5: Line segment attached to two slots

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